MATHEMATICS MODEL PAPER

Part - III : MATHEMATICS – IA

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** sections A, B and C.

SECTION – A

I. Very Short Answer Type Questions : **$10 \times 2 = 20$**

i) Answer ALL the questions.

ii) Each question carries TWO marks.

Q.No : 1 Functions :

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \to B$ is a surjection defined by $f(x) = \cos x$ then find B.
2. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B.
3. If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 3x - 1$, $g(x) = x^2 + 1$, then find (fog) (2).
4. If $f(x) = 2$, $g(x) = x^2$, $h(x) = 2x$ for all $x \in R$, then find $[fo(goh)(x)]$.
5. If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = 4x - 1$ and $g(x) = x^2 + 2$, then find

   (i) (gof) (x) (ii) (gof) $\left(\frac{a+1}{4}\right)$ (iii) (fof) (x) (iv) [go (fof)] (0)
6. Find the inverse function of $f(x) = ax + b$, $(a \neq 0)$; $a, b \in R$
7. $f : R \to R$ defined by $f(x) = \frac{2x+1}{3}$, then this function is injection or not ? Justify.
8. If $f = \{(1, 2), (2, -3), (3, -1)\}$ then find i) 2f ii) 2 + f
9. If $f = \{(4, 5), (5, 6), (6, -4)\}$ and $g = \{(4, -4), (6, 5), (8, 5)\}$, then find i) f + g ii) fg
10. If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 3x - 1$, $g(x) = x^2 + 1$, then find i) (fog)(x) ii) (fog)(2).

Q.No : 2 Functions :

1. Find the domain of $f(x) = \sqrt{9 - x^2}$.
2. Find the domain of $f(x) = \frac{1}{(x^2 - 1)(x + 3)}$.
3. Find the domains of $\sqrt{4x - x^2}$.
4. Find the domains of $f(x) = \log(x^2 - 4x + 3)$

5. Find the domains of $f(x) = \frac{\sqrt{3+x}\,\sqrt{3-x}}{x}$.

6. Find the domain and range of $f(x) = \frac{x}{2-3x}$.

7. Find the domains of $f(x) = \frac{1}{\sqrt{1-x^2}}$.

8. Find the domains of $f(x) = \frac{1}{\log(2-x)}$.

9. Find the domains of $f(x) = \sqrt{x+2} + \frac{1}{\log_{10}(1-x)}$.

10. Find the domain of $f(x) = \sqrt{x^2-25}$.

Q.No : 3 Matrices :

1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find X.

2. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$, then find the values of x, y, z and a.

3. Define trace of matrix and find the trace of $\begin{bmatrix} 1 & 2 & -1/2 \\ 0 & -1 & 2 \\ -1/2 & 2 & 1 \end{bmatrix}$.

4. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$, then find the value of k.

5. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find $A + A'$ and AA'.

6. If $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ then find $2A + B'$ and $3B' - A$.

7. If $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$, then find $(AB')'$.

8. If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew symmetric matrix, then find x.

9. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, then find x.

10. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$ and $X = A + B$ then find X.

Q.No : 4 Matrices :

1. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and det A = 45, then find x.

2. Find the determinant of $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$.

3. Find the determinent of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ where $w^3 = 1$.

4. Find the adjoint and the inverse of the matrix $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

5. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

6. Find the adjoint and the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$.

7. Find the cofactors of 2 and – 5 in the matrix $\begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -2 \\ -4 & -5 & 3 \end{bmatrix}$

8. If $\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$ symmetric (or) skew symmetric ?

9. If $A = \begin{bmatrix} 0 & 4 & -2 \\ -4 & 0 & 8 \\ 2 & -8 & x \end{bmatrix}$ is a skew symmetric matrix and the value of x.

10. Define singular and non - singular matrix.

Q.No : 5 Addition Of Vectors :

1) Find the unit vector in the direction of the sum of the vectors $\bar{a}=2\bar{i}+2\bar{j}-5\bar{k}$ and $\bar{b}=2\bar{i}+\bar{j}+3\bar{k}$.

2. $\bar{a}=2\bar{i}+5\bar{j}+\bar{k}$ and $\bar{b}=4\bar{i}+m\bar{j}+n\bar{k}$ are collinear vectors, then find m and n.

3. If the vectors $3\bar{i}+4\bar{j}+\lambda\bar{k}$ and $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear vectors, then find λ and μ.

4. If $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}$, $\bar{b}=\bar{i}+\bar{j}+\bar{k}$ and $\bar{c}=\bar{j}+2\bar{k}$, find the unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$.

5. If $\overline{OA}=\bar{i}+\bar{j}+\bar{k}$, $\overline{AB}=3\bar{i}-2\bar{j}+\bar{k}$, $\overline{BC}=\bar{i}+2\bar{j}-2\bar{k}$ and $\overline{CD}=2\bar{i}+\bar{j}+3\bar{k}$, then find the vector $\overline{OD}$.

6. If the position vectors of the points A, B and C are $-2\bar{i}+\bar{j}-\bar{k}$, $-4\bar{i}+2\bar{j}+2\bar{k}$ and $6\bar{i}-3\bar{j}-13\bar{k}$ respectively and $\overline{AB}=\lambda\overline{AC}$, then find the value of λ.

7. Write direction ratios of the vector a = i + j – 2k and hence calculate its direction cosines.

8. ABCDE is a pentagon. If the sum of the vectors $\overline{AB}, \overline{AE}, \overline{BC}, \overline{DC}, \overline{ED}$ and $\overline{AC}$ is $\lambda\overline{AC}$, then find the value of λ.

9. Find a vector in the direction of vector a = $\bar{i}-2\bar{j}$ that has magnitude 7 units.

10. Let a, b be non-collinear vector. If α = (x + 4y) a + (2x + y + 1) b and β = (y – 2x + 2) a + (2x – 3y – 1) b are such that 3α = 2β then find x and y.

Q.No : 6 Addition Of Vectors :

1. Find the vector equation of the line passing through the point $2\bar{i}+3\bar{j}+\bar{k}$ and parallel to the vector $4\bar{i}-2\bar{j}+3\bar{k}$.

2. Find the vector equation of the line joining the points $2\bar{i}+\bar{j}+3\bar{k}$ and $-4\bar{i}+3\bar{j}-\bar{k}$.

3. Find the vector equation of the plane passing through the points $\bar{i}-2\bar{j}+5\bar{k}$, $-5\bar{j}-\bar{k}$ and $-3\bar{i}+5\bar{j}$.

4. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

5. Show that the points whose position vectors are $-2\bar{a}+3\bar{b}+5\bar{c}$, $\bar{a}+2\bar{b}+3\bar{c}$, $7\bar{a}-\bar{c}$ are collinear when $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.

6. Find the vector which passes through the points $2\bar{i}+4\bar{j}+2\bar{k}$, $2\bar{i}+3\bar{j}+5\bar{k}$ and parallel to the vector $3\bar{i}-2\bar{j}+\bar{k}$.

7. If a, b, c are the position vectors of the vertices A, B and C respectively of ΔABC, then find the vector equation of the median through the vertex A.

8. OABC is a parallelogram. If $\overline{OA} = \bar{a}$ and $\overline{OC} = \bar{c}$, find the vector equation of the side $\overline{BC}$.

9. If α, β, γ are the angles made by the vector $3\bar{i} - 6\bar{j} + 2\bar{k}$ with the positive directions of the co-ordinate axes, then find cos α, cos β, cos γ.

10. In ΔABC, if 'O' is the circumcentre and it is the orthocentre, then show that $\overline{HA} + \overline{HB} + \overline{HC} + 2\overline{HO}$.

Q.No : 7 Product Of Vectors :

1. Find the angle between the vectors $\bar{i} + 2\bar{j} + 3\bar{k}$ and $3\bar{i} - \bar{j} + 2\bar{k}$.

2. $2\bar{i} + \lambda\bar{j} - \bar{k}$ and $4\bar{i} - 2\bar{j} + 2\bar{k}$ are perpendicular to each other, then find λ.

3. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.

4. If $\bar{a} = 2\bar{i} + 2\bar{j} - 3\bar{k}, \bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, then find the angle between the vectors $2\bar{a} + \bar{b}$ and $\bar{a} + 2\bar{b}$.

5. If $\bar{a} = \bar{i} - \bar{j} - \bar{k}$ and $\bar{b} = 2\bar{i} - 3\bar{j} + \bar{k}$, then find the projection vector of $\bar{b}$ on $\bar{a}$ and its magnitude.

6. $3\bar{i} + 4\bar{j}$ and $-5\bar{i} + 7\bar{j}$ are two sides of a triangle, then find its area.

7. Find the area of the parallelogram whose diagonals are $3\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{i} - 3\bar{j} + 4\bar{k}$.

8. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find p.

9. If θ is the angle between the vectors $\bar{i} + \bar{j}$ and $\bar{j} + \bar{k}$, then find sin θ.

10. Find unit vector perpendicular to both $\bar{a} = 4\bar{i} + 3\bar{j} - \bar{k}$ and $\bar{b} = 2\bar{i} - 6\bar{j} - 3\bar{k}$.

Q.No : 8 Trignometric Ratios Up To Transformations :

1. If tan 20° = λ then show that $\frac{\tan 160° - \tan 110°}{1 + \tan 160° \tan 110°} = \frac{1-\lambda^2}{2\lambda}$.

2. If cos θ + sin θ = $\sqrt{2}$ cos θ, prove that cos θ – sin θ = $\sqrt{2}$ sin θ.

3. If sin α + cosec α = 2, find the value of $\sin^n \alpha + \text{cosec}^n \alpha$; n ∈ Z.

4. Show that sin 780° . sin 480° + cos 240°. cos 300° = 1/2

5. If 3 sin A + 5 cos A = 5, then show that 5 sin A – 3 cos A = ± 3.

6. - Find the period of $f(x) = \cos\left(\frac{4x+9}{5}\right)$.

7. Find the period of $f(x) = \tan(x + 4x + 9x + ... + n^2x)$ (n any positive integer).

8. Find the sine functions whose period is 2/3.

9. If $\sin\theta = -\frac{1}{3}$ and θ does not lie in the third quadrant, find the value of $\cos\theta$, $\cot\theta$.

10. Sketch the graph of the function $\tan x$ between 0 and $\frac{\pi}{4}$.

Q.No : 9 Trignometric Ratios Up To Transformations :

1. Prove that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$.

2. If $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b}$, then prove that $a\sin 2\alpha + b\cos 2\alpha = b$.

3. If $A - B = \frac{3\pi}{4}$, then show that $(1 - \tan A)(1 + \tan B) = 2$.

4. Show that $\cos 42^\circ + \cos 78^\circ + \cos 162^\circ = 0$.

5. Prove that $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$.

6. Find the value of $\tan 56^\circ - \tan 11^\circ - \tan 56^\circ . \tan 11^\circ$.

7. Find the value of $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$.

8. Find the minimum and maximum values of $3\cos x + 4\sin x$

9. Find the range of $7\cos - 24\sin x + 5$

10. If $\sin(\theta + \alpha) = \cos(\theta + \alpha)$, then express $\tan\theta$ in terms of $\tan\alpha$.

Q.No : 10 Hyperbolic Functions :

1. If $\sinh x = 5$, show that $x = \log_e(5 + \sqrt{26})$

2. If $\sinh x = 3$, then show that $x = \log_e(3 + \sqrt{10})$.

3. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$.

4. If $\sinh x = \left(\frac{3}{4}\right)$, find $\cosh(2x)$ and $\sinh(2x)$.

5. If $\cosh x = \frac{5}{2}$, find the values of (i) $\cosh(2x)$ (ii) $\sinh(2x)$.

6. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$, for any $n \in R$

7. Prove that $(\cosh x + \sinh x)^n = \cosh(nx) + \sinh(nx)$, for any $x \in R$

8. For any $x \in R$ prove that $\cosh^4 x - \sinh^4 x = \cosh(2x)$

9. Prove that for any $x \in R$, $\sinh(3x) = 3\sinh x + 4\sinh^3 x$.

10. If $\cosh x = \sec\theta$, then prove that $\tanh^2 \frac{x}{2} = \tan^2 \frac{\theta}{2}$.

SECTION – B

II. Short Answer Type Questions : **4 × 5 = 20**

i) Answer ANY FIVE questions.

ii) Each question carries FOUR marks.

Q.No : 11 Matrices :

1. If $\theta - \phi = \frac{\pi}{2}$, then show that $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} = 0$.

2. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A - 5I = 0$.

3. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then find $A^3 - 3A^2 - A - 3I$.

4. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$.

5. If $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$, then verify that $(AB)' = B'A'$.

6. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then A is invertiable and $A^{-1} = \frac{\text{Adj } A}{\det A}$.

7. Show that $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ is non-singular and find A^{-1}.

8. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then show that $A^{-1} = A'$.

9. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then show that the adjoint of A is 3A'. Find A^{-1}.

10. Find the adjoint and the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

11. Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

Q.No : 12 Addition of Vectors :

1. Let ABCDEF be a regular hexagon with centre 'O'. Show that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$.

2. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the four points $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$, $-3\bar{a} + 2\bar{b} + \bar{c}$ are coplanar.

3. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the four points $6\bar{a} + 2\bar{b} - \bar{c}$, $2\bar{a} - \bar{b} + 3\bar{c}$, $-\bar{a} + 2\bar{b} - 4\bar{c}$, $-12\bar{a} - \bar{b} - 3\bar{c}$ are coplanar.

4. $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors prove that the following four points are coplanar $2\bar{a} + 3\bar{b} - \bar{c}, \bar{a} - 2\bar{b} + 3\bar{c}, 3\bar{a} + 4\bar{b} - 2\bar{c}, \bar{a} - 6\bar{b} + 6\bar{c}$.

5. If $\bar{i}, \bar{j}, \bar{k}$ are unit vectors along the positive directions of the coordinate axes, then show that the four points $4\bar{i} + 5\bar{j} + \bar{k}, -\bar{j} - \bar{k}, 3\bar{i} + 9\bar{j} + 4\bar{k}$ and $-4\bar{i} + 4\bar{j} + 4\bar{k}$ are coplanar.

6. If α, β, γ are the angles made by the vector $3\bar{i} - 6\bar{j} + 2\bar{k}$ with the positive directions of the co-ordinate axes, then find $\cos\alpha, \cos\beta, \cos\gamma$.

7. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.

8. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vector, then find the point of intersection of the line passing through the points $2\bar{a} + 3\bar{b} - \bar{c}, 3\bar{a} + 4\bar{b} - 2\bar{c}$ with the line joining the points $\bar{a} - 2\bar{b} + 3\bar{c}, \bar{a} - 6\bar{b} + 6\bar{c}$.

9. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}, \bar{a} + 2\bar{b} - 5\bar{c}$ intersect at the point $-4\bar{c}$ when $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.

10. Is the triangle formed by the vectors $3\bar{i}+5\bar{j}+2\bar{k}$, $2\bar{i}-3\bar{j}-5\bar{k}$ and $-5\bar{i}-2\bar{j}+3\bar{k}$ equilateral?

Q.No : 13 Product of Vectors :

1. Find the area of the triangle whose vertices are A(1, 2, 3), B(2, 3, 1) and C(3, 1, 2).
2. If $\bar{a}=2\bar{i}+\bar{j}-\bar{k}, \bar{b}=-\bar{i}+2\bar{j}-4\bar{k}$ and $\bar{c}=\bar{i}+\bar{j}+\bar{k}$, then find $(\bar{a}\times\bar{b}).(\bar{b}\times\bar{c})$.
3. If $\bar{a}=\bar{i}-2\bar{j}-3\bar{k}, \bar{b}=2\bar{i}+\bar{j}-\bar{k}$ and $\bar{c}=\bar{i}+3\bar{j}-2\bar{k}$, verify that $\bar{a}\times(\bar{b}\times\bar{c})\neq(\bar{a}\times\bar{b})\times\bar{c}$.
4. Find the volume of the tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, – 1, 0) and (– 1, 0, 1).
5. Find the volume of the tetrahedron having the edges $\bar{i}+\bar{j}+\bar{k}$; $\bar{i}-\bar{j}$ and $\bar{i}+2\bar{j}+\bar{k}$.
6. Find the unit vector perpendicular to the plane passing through the points (1, 2, 3), (2, – 1, 1) and (1, 2, – 4).
7. Let $\bar{a}=2i+j-2k, \bar{b}=i+j$. If $\bar{c}$ is a vector such that $\bar{a}.\bar{c}=|\bar{c}|, |\bar{c}-\bar{a}|=2\sqrt{2}$ and the angle between $\bar{a}\times\bar{b}$ and $\bar{c}$ is 30°, then find the value of $|(\bar{a}\times\bar{b})\times\bar{c}|$.
8. If $\bar{a}=2\bar{i}+3\bar{j}+4\bar{k}, \bar{b}=\bar{i}+\bar{j}-\bar{k}$ and $\bar{c}=\bar{i}-\bar{j}+\bar{k}$, then compute $\bar{a}\times(\bar{b}\times\bar{c})$ and verify that it is perpendicular to $\bar{a}$.
9. Find λ in order that the four points A(3, 2, λ), B(4, λ, 5), C(4, 2, – 2) and D(6, 5, – λ) be coplanar.
10. If $\bar{a}+\bar{b}+\bar{c}=\bar{0}$, $|\bar{a}|=3, |\bar{b}|=5$ and $|\bar{c}|=7$, then find the angle between $\bar{a}$ and $\bar{b}$.

Q.No : 14 Trignometric Ratios Up To Transformations :

1. Prove that $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=\frac{3}{2}$
2. Show that $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$.
3. Prove that $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$
4. If A is not an integral multiple of π, prove that cos A. cos 2A. cos 4A . cos 8A = $\frac{\sin 16A}{16\sin A}$ and hence deduce that $\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}=\frac{1}{16}$.
5. If $0<A<B<\frac{\pi}{4}$ and $\sin(A+B)=\frac{24}{25}$ and $\cos(A-B)=\frac{4}{5}$, then find the value of tan 2A.
6. If a, b, c are non zero real numbers and α, β are the solutions of $a\cos\theta+b\sin\theta=c$ then show that (i) $\sin\alpha+\sin\beta=\frac{2bc}{a^2+b^2}$ and (ii) $\sin\alpha+\sin\beta=\frac{c^2-a^2}{a^2+b^2}$.

7. Prove that $\sin A \cdot \sin\left(\frac{\pi}{3}+A\right)\sin\left(\frac{\pi}{3}-A\right)=\frac{1}{4}\sin 3A$ and hence deduce that $\sin 20° \cdot \sin 40° \cdot \sin 60° \cdot \sin 80° = \frac{3}{16}$.

8. If none of the denominators is zero, prove that $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n = 2\cot^n\left(\frac{A-B}{2}\right)$, if n is even 0, if n is odd.

9. Prove that $\frac{\mathrm{Cos}^3\theta-\mathrm{Cos}3\theta}{\mathrm{Cos}\theta}+\frac{\mathrm{Sin}^3\theta+\mathrm{Sin}3\theta}{\mathrm{Sin}\theta}=3.$

10. If θ is not an integral multiple of $\frac{\pi}{2}$, prove that $\tan\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta=\cot\theta$.

Q.No : 15 Trignometric Equations :

1. Solve $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$.

2. Solve $4\cos^2\theta+\sqrt{3}=2(\sqrt{3}+1)\cos\theta$.

3. Find the general solution of the equation $1+\sin^2\theta=3\sin\theta\cdot\cos\theta$.

4. Solve $\sqrt{2}(\sin x+\cos x)=\sqrt{3}$.

5. Solve $\sqrt{3}\sin\theta-\cos\theta=\sqrt{2}$.

6. Solve $\sin x+\sqrt{3}\cos x=\sqrt{2}$.

7. Solve $\cot^2 x-(\sqrt{3}+1)\cot x+\sqrt{3}=0,\ \left(0<x<\frac{\pi}{2}\right)$.

8. If θ_1, θ_2 are solutions of the equation $a\cos 2\theta+b\sin 2\theta=c$, $\tan\theta_1\neq\tan\theta_2$ and $a+c\neq 0$, then find the values of i) $\tan\theta_1+\tan\theta_2$ ii) $\tan\theta_1 \cdot \tan\theta_2$.

9. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$, then prove that $\cos\left(\theta-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}$.

10. If α, β are the solutions of $a\cos\theta+b\sin\theta=c$, then show that

i) $\sin\alpha+\sin\beta=\frac{2bc}{a^2+b^2}$ ii) $\sin\alpha \cdot \sin\beta=\frac{c^2-a^2}{a^2+b^2}$

Q.No : 16 Inverse Trignometric Equations :

1. Prove that $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{7}{25}=\sin^{-1}\frac{117}{125}$.

2. Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{36}{85}\right)$.

3. Prove that $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}$.

4. Prove that $\text{Tan}^{-1}\frac{1}{2} + \text{Tan}^{-1}\frac{1}{5} + \text{Tan}^{-1}\frac{1}{8} = \frac{\pi}{4}$.

5. Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{3}{\sqrt{34}}\right) = = \tan^{-1}\left(\frac{27}{11}\right)$

6. Prove that $\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} - \tan^{-1}\frac{2}{9} = 0$.

7. Show that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(2\tan^{-1}\frac{3}{4}\right)$.

8. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then prove that $x + y + z = xyz$.

9. Solve $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$.

10. If $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = \pi$ then prove that $p^2 + q^2 + r^2 + 2pqr = 1$.

Q.No : 17 (Properties of triangles) :

1. Show that $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$.

2. Prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$.

3. In a ΔABC, show that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$.

4. In a ΔABC, If $a = (b - c)\sec\theta$, then prove that $\sin\theta = \frac{2\sqrt{bc}}{b+c}\cos\left(\frac{A}{2}\right)$.

5. Prove that $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} = \frac{bc + ca + ab - s^2}{\Delta}$.

6. In a ΔABC, Prove that $(r_1 + r_2)\sec^2\frac{C}{2} = (r_2 + r_3)\sec^2\frac{A}{2} = (r_3 + r_1)\sec^2\frac{B}{2}$.

7. Show that $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + \frac{r}{2R}$.

8. In a ΔABC, If $a : b : c = 7 : 8 : 9$, find $\cos A : \cos B : \cos C$.

9. If p_1, p_2, p_3 are the altitudes of ΔABC, then show that $\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=\frac{\cot A+\cot B+\cot C}{\Delta}$

10. Show that $a^2 \cot A + b^2 \cot B + c^2 \cot C = \frac{abc}{R}$.

SECTION – C

III. Long Answer Type Questions : $5 \times 7 = 35$

i) Answer ANY FIVE questions.

ii) Each question carries SEVEN marks.

Q.No : 18 Functions :

1. If $f : A \to B$, $g : B \to C$ be bijections, then prove that $gof : A \to C$ is a bijection.
2. If $f : A \to B$, $g : B \to C$ be bijections, then prove that $(gof)^{-1} = f^{-1}og^{-1}$.
3. Let $f : A \to B$, I_A and I_B are identity functions on A and B respectively. Then prove that $foI_A = f = I_Bof$.
4. If $f : A \to B$ be a bijection, then prove that $fof^{-1} = I_B$ and $f^{-1}of = I_A$.
5. If $f : A \to B$, $g : B \to A$ are two functions such that $gof = I_A$ and $fog = I_B$, then $f : A \to B$ is a bijection and $f^{-1} = g$.

Q.No : 19 Mathematical Induction :

1. Using the principles of Mathematical induction, prove that 1.2.3 + 2.3.4 + 3.4.5 + ... upto n terms = $\frac{n(n+1)(n+2)(n+3)}{4}$.
2. Using the principles of Mathematical induction, prove that $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots\dots$ upto n terms = $\frac{n(n+1)^2(n+2)}{12}$.
3. Using the principles of Mathematical induction prove that $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$.
4. Using the principles of Mathematical induction prove that $\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\frac{1}{7\cdot10}+$ .... upto n terms $=\frac{n}{3n+1}\ \forall n \in N$.
5. By Mathematical Induction $\forall n \in N$, prove that 2.3 + 3.4 + 4.5 + ... upto n terms $= \frac{n(n^2+6n+11)}{3}$.

6. Using the principles of mathematical induction, prove that

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\text{............ upto n terms}=\frac{n}{24}(2n^2+9n+13).$$

7. Using the principles of mathematical induction, prove the $a+(a+d)+(a+2d)+$ .... upto n terms $=\frac{n}{2}[2a+(n-1)d]$.

8. Prove by Mathematical Induction $a+ar+ar^2+$ .... upto n terms $=a\left(\frac{r^n-1}{r-1}\right)$.

9. By Mathematical Induction, Sshow that $49^n+16n-1$ is divisible by 64 for all positive integer n.

10. Show that $3\cdot5^{2n+1}+2^{3n+1}$ is divisible by 17 for all positive integers n.

Q.No : 20 Matrices :

1. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2=\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}=(a^3+b^3+c^3-3abc)^2$.

2. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}=2(a+b+c)^3$

3. Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}=(a+b+c)^3$.

4. Show that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}=2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

5. Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}=(a-b)(b-c)(c-a)(ab+bc+ca)$.

6. Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}=abc(a-b)(b-c)(c-a)$.

7. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0$ then show that $abc = -1$.

8. Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$.

9. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.

10. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then A is invertiable and $A^{-1} = \frac{\text{Adj } A}{\det A}$.

Q.No : 21 Matrices :

1. Solve the equations by using Cramer's Rule $2x - y + 3z = 9, x + y + z = 6, x - y + z = 2$.
2. Solve the equations $3x + 4y + 5z = 18, 2x - y + 8z = 13, 5x - 2y + 7z = 20$ by Cramer's rule.
3. Solve the equations $2x - y + 3z = 9, x + y + z = 6, x - y + z = 2$ by Cramer's Rule.
4. Solve the equations $3x + 4y + 5z = 18, 2x - y + 8z = 13$ and $5x - 2y + 7z = 20$ by Matrix inversion method.
5. Solve the equations $x + y + z = 9, 2x + 5y + 7z = 52, 2x + y - z = 0$ by matrix inversion method.
6. Solve the equations $2x - y + 3z = 9, x + y + z = 6, x + y + z = 2$ by Gauss - Jordan method.
7. Solve the equations $x + y + z = 9, 2x + 5y + 7z = 52, 2x + y + z = 0$ by Gauss - Jordan method.
8. Solve the equations $3x + 4y + 5z = 18, 2x - y + 8z = 13, 5x - 2y + 7z = 20$ by Gauss - Jordan method.
9. Show that the following system of equations is consistent and solve it completely $x + y + z = 3, 2x + 2y = 3, x + y - z = 1$.
10. Examine whether the following system of equations are consistents in consistent and if consistent, find the complete solution. $x + y + z = 6, x + 2y + 3z = 10, x + 2y + 4z = 1$

Q.No : 22 Product of Vectors :

1. If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}, \bar{b} = 2\bar{i} + \bar{j} + \bar{k}, \bar{c} = \bar{i} + \bar{j} + 2\bar{k}$, then find $|(\bar{a} \times \bar{b}) \times \bar{c}|$ and $|\bar{a} \times (\bar{b} \times \bar{c})|$.
2. If $\bar{a} = \bar{i} - 2\bar{j} + \bar{k}, \bar{b} = 2\bar{i} + \bar{j} + \bar{k}, \bar{c} = \bar{i} + 2\bar{j} - \bar{k}$, then find $\bar{a} \times (\bar{b} \times \bar{c})$ and $|(\bar{a} \times \bar{b}) \times \bar{c}|$.
3. Find the shortest distance between the skew lines $\bar{r} = (6\bar{i} + 2\bar{j} + 2\bar{k}) + t(\bar{i} - 2\bar{j} + 2\bar{k})$ and $\bar{r} = (-4\bar{i} - \bar{k}) + s(3\bar{i} - 2\bar{j} - 2\bar{k})$.

4. If A = (1, – 2, – 1), B = (4, 0, – 3), C = (1, 2, – 1) and D = (2, – 4, – 5), find the distance between AB and CD.

5. If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, then prove that

i) $(\bar{a}\times\bar{b})\times\bar{c} = (\bar{c}.\bar{a})\bar{b} - (\bar{c}.\bar{b})\bar{a}$ ii) $\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}$

6. If $\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$, $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$, $\bar{c} = -\bar{i} + \bar{j} - 4\bar{k}$ and $\bar{d} = \bar{i} + \bar{j} + \bar{k}$, then compute $\left|(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})\right|$.

7. Prove by vector methods the angle between the two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

8. Find the equation of the plane passing through the points A = (2, 3, – 1), B = (4, 5, 2) and C = (3, 6, 5).

9. If $\bar{a} = \bar{i} - \bar{j} + \bar{k}, \bar{b} = 2\bar{i} - \bar{j} + 3\bar{k}, \bar{c} = \bar{i} - \bar{j}$ and $\bar{d} = 6\bar{i} + 2\bar{j} + 3\bar{k}$. Express $\bar{d}$ interms of $\bar{b}\times\bar{c}, \bar{c}\times\bar{a}$ and $\bar{a}\times\bar{b}$.

10. If $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}, \bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then find $(\bar{a}\times\bar{b}).(\bar{b}\times\bar{c})$.

Q.No : 23 Trignometric Ratios Up to Transformations :

1. If A + B + C = 180° prove that cos 2A + cos 2B + cos 2C = – 4 cos A cos B cos C – 1.

2. In triangle ABC, prove that $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\sin\frac{C}{2}$

3. In triangle ABC, prove that $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}\right)$

4. In triangle ABC, prove that $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{\pi - A}{4}\sin\frac{\pi - B}{4}\sin\frac{\pi - C}{4}$

5. In triangle ABC, prove that $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$

6. If A + B + C = 2S, then prove that $\sin(S - A) + \sin(S - B) + \sin C = 4\cos\frac{S - A}{2}\cos\frac{S - B}{2}\sin\frac{C}{2}$.

7. In triangle ABC, prove that $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$

8. In triangle ABC, prove that $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A\sin B\cos C$.

Q.No : 24 Properties of Triangles :

1. If $r_1 = 2, r_2 = 3, r_3 = 6$ and $r = 1$, then prove that a = 3, b = 4 and c = 5.

2. In ΔABC, if $r_1 = 8, r_2 = 12, r_3 = 24$; find a, b, c.

3. If a = 13, b = 14, c = 15, show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.

4. In Δ ABC, then show that $r_1 + r_2 + r_3 - r = 4R$.

5. Show that $r + r_3 + r_1 - r_2 = 4R \cos B$ in a triangle ABC.

6. In ΔABC, then show that $r + r_1 + r_2 - r_3 = 4R \cos C$.

7. If $\sin\theta = \frac{a}{b+c}$, then show that $\cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\left(\frac{A}{2}\right)$.

8. If $a = (b - c)\sec\theta$, prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\left(\frac{A}{2}\right)$.

9. In ΔABC, then show that $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{r} - \frac{1}{2R}$.

10. In ΔABC then prove that $a\cos^2\frac{A}{2} + b\cos^2\frac{B}{2} + c\cos^2\frac{C}{2} = s + \frac{\Delta}{R}$.

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MATHEMATICS MODEL PAPER

MATHEMATICS – IB

Time : 3 Hours] [Max. Marks : 75

Note : This question paper consists of **three** sections **A, B** and **C**.

SECTION – A

I. Very Short Answer Type Questions : $10 \times 2 = 20$

i) Answer ALL the questions.

ii) Each question carries TWO marks.

Q.No : 1 Straight Line :

1. Find the value of x, if the slope of the line passing through (2, 5) and (x, 3) is 2.
2. Find the equation of the straight line passing through (– 4, 5) and cutting off equal and non-zero intercepts on the coordinate axes.
3. Find the condition for the points (a, 0), (h, k) and (0, b), where $ab \neq 0$, to be collinear.
4. If the product of the intercepts made by the straight line $x \tan\alpha + y \sec\alpha = 1 \left(0 \le \alpha < \frac{\pi}{2}\right)$ on the coordinate axes is equal to $\sin\alpha$, find α.
5. Find the equation of the straight line passing through the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$.
6. Find the value of y, if the line joining the points (3, y) and (2, 7) is parallel to the line joining the points (–1, 4) and (0, 6).
7. If the area of the triangle formed by the straight lines, $x = 0$, $y = 0$ and $3x + 4y = a (a > 0)$ is '6'. Find the value of 'a'.
8. Transform the equation $x + y + 1 = 0$ into normal form.
9. Find the equation of the straight line passing through (–2, 4) and making non-zero intercepts whose sum is zero.
10. Transform the equation into the form $L_1 + \lambda L_2 = 0$ and find the point of concurrency of the family of straight lines represented by the equations $(2 + 5k)x - 3(1 + 2k)y + (2 - k) = 0$.

Q.No : 2 Straight Line :

1. Find the distance between the parallel lines $5x - 3y - 4 = 0$, $10x - 6y - 9 = 0$.
2. Find the equation of the straight line parallel to the line $2x + 3y + 7 = 0$ and passing through the point (5, 4).
3. Find the angle which the straight line $y = \sqrt{3}x - 4$ makes with the y-axis.
4. If $2x - 3y - 5 = 0$ is the perpendicular bisector of the line segment joing (3, – 4) and (α, β) find $\alpha + \beta$.
5. If A(10, 4), B(–4, 9) and C(–2, –1) are the vertices of a triangle. Find the equation of the median through 'A'.

6. Find the area of the triangle formed by the following straight lines and the coordinates axes $x \cos\alpha + y \sin\alpha = p$ $(p > 0)$.
7. Find the value of p, if the straight lines $x + p = 0$, $y + 2 = 0$ and $3x + 2y + 5 = 0$ are concurrent.
8. Find the value of k, if the straight lines $y - 3kx + 4 = 0$ and $(2k - 1)x - (8k - 1)y - 6 = 0$ are perpendicular.
9. Find the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ $(a \neq \pm b)$.
10. If A(10, 4), B(–4, 9) and C(–2, –1) are the vertices of a triangle. Find the equation of the altitude through B.

Q.No : 3 Three Dimensional Coordinates :

1. Find the coordinates of the vertex 'C' of ΔABC if its centroid is the origin and the vertices A, B are (1, 1, 1) and (– 2, 4, 1) respectively.
2. If (3, 2, – 1), (4, 1, 1) and (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex.
3. Find the ratio in which the XZ-plane divides the line joining A(– 2, 3, 4) and B(1, 2, 3).
4. Find the ratio in which XY-plane divides the line joining A(2, 4, 5) and B(3, 5, – 4). Also find the point of intersection.
5. Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, –1), (3, 6, –1) and (4, 5, 1).
6. Show that the points A(1, 2, 3), B(7, 0, 1) and C(– 2, 3, 4) are collinear.
7. Find the centroid of the tetrahedron whose vertices are (2, 3, – 4), (– 3, 3, – 2), (– 1, 4, 2) and (3, 5, 1).
8. By section formula find the point which divides the line joining the points A(2, – 3, 1) and B(3, 4, – 5) in the ratio 1 : 3.
9. Show that the points (1, 2, 3), (2, 3, 1) and (3, 1, 2) form an equilateral triangle.
10. Find the ratio in which the point (6, – 17, – 4) divides the line segment joining (2, 3, 4) (3, – 2, 2).

Q.No : 4 The Plane :

1. Write the equation of the plane $4x - 4y + 2z + 5 = 0$ in the intercept form.
2. Find the intercepts of the plane on the coordinate axes $4x + 3y - 2z + 2 = 0$.
3. Find the equation of the plane whose intercepts on x, y, z - axes are 1, 2, 4 respectively.
4. Find the directions of the normal to the plane $x + 2y + 2z - 4 = 0$.
5. Reduce the equation $x + 2y - 3z - 6 = 0$ of the plane to the normal form.
6. Find the angle between the planes $x + 2y + 2z - 5 = 0$ and $3x + 3y + 2z - 8 = 0$.
7. Find the angle between the planes, $2x - y + z = 6$ and $x + y + 2z = 7$.
8. Find the equation of the plane if the foot of the perpendicular from origin to the plane is (1, 3, – 5).
9. Find the constant k so that the planes $x - 2y + kz = 0$ and $2x + 5y - z = 0$ are at right angles.
10. Find the equation to the plane parallel to the ZX – plane and passing through (0, 4, 4).
11. Find the equation of the plane passing through the point (1, 1, 1) and parallel to the plane $x + 2y + 3z - 7 = 0$.

Q.No : 5 Limits And Continuity :

1. Find $\underset{x\to 0}{\text{Lt}}\left(\frac{\sqrt{1+x}-1}{x}\right)$.
2. Evaluate $\underset{x\to a}{\text{Lt}}\frac{\tan(x-a)}{x^2-a^2}$.
3. Compute $\underset{x\to 0}{\text{Lt}}\frac{\sin ax}{x\cos x}$.
4. Evaluate $\underset{x\to 1}{\text{Lt}}\frac{\sin(x-1)}{x^2-1}$.
5. Compute $\underset{x\to 0}{\text{Lt}}\frac{e^{7x}-1}{x}$.
6. Compute $\underset{x\to 0}{\text{Lt}}\frac{\log_e(1+5x)}{x}$.
7. Evaluate $\underset{x\to \frac{\pi}{2}}{\text{Lt}}\frac{\cos x}{\left(x-\frac{\pi}{2}\right)}$.
8. Show that $\underset{x\to 2^-}{\text{Lt}}\frac{|x-2|}{x-2}=-1$.
9. Show that $\underset{x\to 0^+}{\text{Lt}}\left(\frac{2|x|}{x}+x+1\right)=3$.
10. Find the right and the left limits of the function $f(x)=\begin{cases}x/2 & (x<2)\\ x^3/3 & (x\ge 2)\end{cases}$ at the point a = 2.

Q.No : 6 Limits And Continuity :

1. Compute $\underset{x\to\infty}{\text{Lt}}(\sqrt{x^2+x}-x)$.
2. Compute $\underset{x\to\infty}{\text{Lt}}(\sqrt{x+1}-\sqrt{x})$.
3. Evaluate $\underset{x\to 0}{\text{Lt}}\left(\frac{\cos ax-\cos bx}{x^2}\right)$.
4. Evaluate $\underset{x\to a}{\text{Lt}}\frac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}$.
5. Evaluate $\underset{x\to 0}{\text{Lt}}\frac{\sin(a+bx)-\sin(a-bx)}{x}$.
6. Compute $\underset{x\to 0}{\text{Lt}}\frac{a^x-1}{b^x-1}$, $(a>0), (b>0), (b\ne 1)$.
7. Evaluate $\underset{x\to -\infty}{\text{Lt}}\left(\frac{2x+3}{\sqrt{x^2-1}}\right)$.
8. Evaluate $\underset{x\to 0}{\text{Lt}}\frac{1-\cos 2mx}{\sin^2 nx}$ $(m, n\in z)$.
9. Compute $\underset{x\to 0}{\text{Lt}}\frac{\sin(\pi\cos^2 x)}{x^2}$.
10. Evaluate $\underset{x\to\infty}{\text{Lt}}\frac{8|x|+3x}{3|x|-2x}$.

Q.No : 7 Differentiation :

1. If $y=\cos(\log(\cot x))$, then find $\frac{dy}{dx}$.
2. Find the derivative of $20^{\log(\tan x)}$ with reference to x.
3. Find the derivative of 7^{x^3+3x} with reference to x.
4. Find the derivatives of $\sin^{-1}\sqrt{x}$ w.r.t. x.
5. Find the derivatives of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. x.
6. Find the derivative of $\sec(\sqrt{\tan x})$ w.r.t. x find $f^1(x)$.
7. If $y=\log(\sin(\log x))$, then find $\frac{dy}{dx}$.
8. Find the derivative of $\log(\tan 5x)$ w.r.t. x.
9. Find the derivative of $\frac{\sin x}{1+\cos x}$ w.r.t. x.
10. If $y=x^2e^x\sin x$, find $\frac{dy}{dx}$.
11. Find the derivative of $f(x)=e^x(x^2+1)$.
12. If $x=a\cos^3 t$, $y=a\sin^3 t$, then find $\frac{dy}{dx}$.

Q.No : 8 Differentiation :

1. Find the derivative of $\log(\sin^{-1}(e^x))$ w.r.t.x.
2. If $x = \tan e^{-y}$ then show that $\frac{dy}{dx} = \frac{-e^y}{1+x^2}$.
3. Find the derivative of $\operatorname{cosec}^{-1}(e^{2x+1})$ w.r.t. x.
4. If $y = \{\cot^{-1}(x^3)\}^2$, then find $\frac{dy}{dx}$.
5. Find the derivative of $\tan^{-1}\left(\frac{a-x}{1+ax}\right)$ w.r.t.x.
6. Find the derivative of $\sin h^{-1}\left(\frac{3x}{4}\right)$ w.r.t. x.
7. Find the derivative of $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ find $\frac{dy}{dx}$.
8. Differentiation $f(x) = e^x$ with respect to $g(x) = \sqrt{x}$.
9. If $y = x^x$ $(x > 0)$, then $\frac{dy}{dx} = x^x(1 + \log x)$.
10. If $x^3 + y^3 - 3axy = 0$, then find $\frac{dy}{dx}$.

Q.No : 9 Errors And Approximations :

1. Find Δy and dy if $y = 5x^2 + 6x + 6$, $x = 2$, $\Delta x = 0.001$.
2. Find Δy and dy if $y = f(x) = x^2 + x$, $x = 10$ and $\Delta x = 0.1$.
3. Find Δy and dy if $y = x^2 + 3x + 6$, $x = 10$, $\Delta x = 0.01$.
4. Find Δy and dy if $y = e^x + x$, $x = 5$ and $\Delta x = 0.02$.
5. Find the approximatioin of $\sqrt[3]{65}$.
6. Find the approximation of $\sqrt{82}$.
7. If the increase in the side of a square is 2%, then find the approximate percentage of increase in its area.
8. If the increase in the side of a square is 4%, then find the approximate percentage of increase in the area of the square.
9. If the radius of a sphere is increased from 7cm to 7.02 cm then find the approximate increase in the volume of the sphere.
10. The side of a square is increased from 3 cm to 3.01 cm. Find the approximate increase in the area of the square.

Q.No : 10 Rolle's Theorem Ang Lagrange's Mean Value Theorem :

1. Verify Rolle's theorem for the function $y = f(x) = x^2 + 4$ in $[-3, 3]$.
2. Find the value of 'c' Rolle's theorem for the function $x^2 - 1$ on $[2, 3]$.
3. Verify Rolle's theorem for the following function $x^2 - 1$ on $[-1, 1]$.
4. It is given that Rolle's theroem holds for the function $f(x) = x^3 + bx^2 + ax$ on $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

5. Let $f(x) = (x-1)(x-2)(x-3)$. Prove that there is more than one 'c' in $(1, 3)$ such that $f'(c) = 0$.
6. Find a point on the graph of the curve $y = x^3$ where the tangent is parallel to the chord joining $(1, 1)$ and $(3, 27)$.
7. On the curve $y = x^2$, find a point at which the tangent is parallel to the chord joining $(0, 0)$ and $(1, 1)$.
8. Show that there is no real number k, for which the equation $x^2 - 3x + k = 0$ has two distinct roots in $[0, 1]$.
9. Verify the Rolle's theorem for the function $(x^2-1)(x-2)$ on $[-1, 2]$. Find a point in the interval where the derivative vanishes.
10. Find 'c' so that $f'(c) = \frac{f(b)-f(a)}{b-a}$ in the cases $f(x) = x^3 - 3x - 1$; $a = \frac{-11}{7}$, $b = \frac{13}{7}$.

SECTION – B

II. Short Answer Type Questions : **$4 \times 5 = 20$**

i) Answer ANY FIVE questions.

ii) Each question carries FOUR marks.

Q.No : 11 Locus :

1. Find the equation of locus of P, if the line segment joining $(2, 3)$ and $(-1, 5)$ subtends a right angle at P.
2. The ends of the hypotenuse of a right angled triangle are $(0, 6)$ and $(6, 0)$. Find the equation of locus of its third vertex.
3. $A(1, 2)$, $B(2, -3)$ and $C(-2, 3)$ are three points. A point P moves such that $PA^2 + PB^2 = 2PC^2$. Show that the equation to the locus of P is $7x - 7y + 4 = 0$.
4. $A(5, 3)$ and $B(3, -2)$ are two fixed points. Find the equation of the locus of P, so that the area of triangle PAB is 9 sq.units.
5. $A(2, 3)$ and $B(-3, 4)$ are two given points. Find the equation of locus of P. So that the area of the triangle PAB is 8.5 sq. units.
6. Find the equation of the locus of P, if the ratio of the distances from P to $A(5, -4)$ and $B(7, 6)$ is $2 : 3$.
7. Find the equation of the locus of P, if $A = (2, 3)$, $B = (2, -3)$ and $PA + PB = 8$.
8. Find the equation of the locus of a point, the sum of whose distances from $(0, 2)$ and $(0, -2)$ is 6.
9. Find the equation of the locus of P, if $A = (4, 0)$, $B = (-4, 0)$ and $|PA - PB| = 4$.
10. Find the equation of the locus of a point, the difference of whose distances from $(-5, 0)$ and $(5, 0)$ is 8.

Q.No : 12 Transformation of Axes :

1. When the origin is shifted to $(-1, 2)$ by the translation of axes, find the transformed equation of $2x^2 + y^2 - 4x + 4y = 0$.
2. When the origin is shifted to the point $(2, 3)$, the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve.
3. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$.

4. When the axes are rotated through an angle α, find the transformed equation of, $x \cos\alpha + y \sin\alpha = P$

5. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2 + 10xy + 3y^2 = 9$.

6. When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

7. Prove that the angle of rotation of the axes to eliminate xy term from the equation $ax^2 + 2hxy + by^2 = 0$ is $\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$ where $a \neq b$ and $\frac{\pi}{4}$ if $a = b$.

8. When the origin is shifted to the point $(3, -4)$, the transformed equation of a curve is $x^2 + y^2 = 4$. Find the original equation of the curve.

9. When the origin is shifted to $(-1, 2)$ by the translation of axes, find the transformed equation of $2x^2 + y^2 - 4x + 4y = 0$.

10. When the origin is shifted to $(-1, 2)$ by the translation of axes find the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$.

Q.No : 13 Straight Line :

1. Transform the equation $\frac{x}{a} + \frac{y}{b} = 1$ into the normal form when $a > 0$ and $b > 0$. If the perpendicular distance of the straight line from the origin is p, deduce that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

2. Find the value of k, if the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent.

3. Show that the lines $2x + y - 3 = 0$, $3x + 2y - 2 = 0$ and $2x - 3y - 23 = 0$ are concurrent and find the point of concurrent.

4. If the straight lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$.

5. Find the value of p, if the lines $3x + 4y = 5$, $2x + 3y = 4$, $px + 4y = 6$ are concurrent.

6. Find the equation of 'k', if the angle between the straight lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ is 45°.

7. $x - 3y - 5 = 0$ is the perpendicular bisector of the line segment joining the points A, B. If $A = (-1, -3)$ find the coordinates of B.

8. A straight line passing through $A(1, -2)$ makes an angle $\tan^{-1}\left(\frac{4}{3}\right)$ with the positive direction of the X - axis in the anti-clockwise sense. Find the points on the straight line whose distance from A is 5.

9. Find the points on the line $3x - 4y - 1 = 0$ which are at a distance of 5 units from the point $(3, 2)$.

10. A straight line through $Q(\sqrt{3}, 2)$ makes an angle $\frac{\pi}{6}$ with the positive direction of the X - axis. If the straight line intersects the line $\sqrt{3}x - 4y + 8 = 0$ at P, find the distance PQ.

11. A straight line with slope 1 passes through $Q(-3, 5)$ meets the line $x + y - 6 = 0$ at P. Find the distance PQ.

Q.No : 14 Limits And Continuity :

1. Compute $\lim_{x \to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$.

2. Compute $\lim_{x \to a} \frac{x \sin a - a \sin x}{x - a}$.

3. Compute $\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2}$.

4. Check the continuity of f given by $f(x) = \begin{cases} \frac{x^2 - 9}{x^2 - 2x - 3} & \text{if } 0 < x < 5 \text{ and } x \neq 3 \\ 1.5 & \text{if } x = 3 \end{cases}$ at the point 3.

5. Check the continuity of the following function at 2.

$$f(x) = \begin{cases} \frac{1}{2}(x^2 - 4) & \text{if } 0 < x < 2 \\ 0, & \text{if } x = 2 \\ 2 - 8x^{-3}, & \text{if } x > 2 \end{cases}$$

6. Show that $f(x) = \begin{cases} \frac{\cos ax - \cos bx}{x^2} & \text{if } x \neq 0 \\ \frac{1}{2}(b^2 - a^2) & \text{if } x = 0 \end{cases}$ where a and b are real constants, is continuous at '0'.

7. Find real constants a, b so that the function f given by $f(x) = \begin{cases} \sin x & \text{if } x \leq 0 \\ x^2 + a & \text{if } 0 < x < 1 \\ bx + 3 & \text{if } 1 \leq x \leq 3 \\ -3 & \text{if } x > 3 \end{cases}$ is continuous on R.

8. Check the continuity of 'f' given by $f(x) = \begin{cases} 4 - x^2, & \text{if } x \leq 0 \\ x - 5, & \text{if } 0 < x \leq 1 \\ 4x^2 - 9, & \text{if } 1 < x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}$ at points x = 0, 1, 2.

9. Is f defined by $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$, continuous on '0' ?

10. If f, given by $f(x) = \begin{cases} k^2x - k & \text{if } k \geq 1 \\ 2 & \text{if } k < 1 \end{cases}$ is a continuous function on R, then find the values of k.

Q.No : 15 Differentiation :

1. Find the derivative of x^3 from the first principle.
2. Find the derivative of sin 2x from the first principle.
3. Find the derivative of cos ax from the first principle.

4. Find the derivative of tan 2x from the first principle.

5. Find the derivative of cot x from the first principle.

6. Find the derivative of sec 3x from the first principle.

7. Find the derivative of x sin x from the first principle.

8. Find the derivative of $\cos^2 x$ from the first principle.

9. If $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$ then find $\frac{dy}{dx}$.

10. If $ay^4 = (x + b)^5$ then $5yy'' = (y')^2$

Q.No : 16 Tangent and Normal :

1. Find the equations of tangent and normal to the curve $xy = 10$ at $(2, 5)$.
2. Show that the tangent at any point 'θ' on the curve $x = c \sec\theta$, $y = c \tan\theta$ is $y \sin\theta = x - c \cos\theta$.
3. Find the equations of the tangents to the curve $y = 3x^2 - x^3$, where it meets the X - axis.
4. Find the equations of tangent and normal to the curve $x = \cos t$, $y = \sin t$ at $t = \frac{\pi}{4}$.
5. Find the equations of tangent and normal to the curve $y = x^3 + 4x^2$ at $(-1, 3)$.
6. Find the equation of tangent and normal to the curve $y = 2.\ e^{-x/3}$ at the point where the curve meets the y-axis.
7. Find the lengths of sub tangent and sub normal at a point on the curve $y = b \sin\left(\frac{x}{a}\right)$.
8. Show that at any point (x, y) on the curve $y = be^{x/a}$, the length of the sub tangent is a constant and the length of the sub normal is $\frac{y^2}{a}$.
9. Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.
10. Show that the equation of the tangent to the curve is $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2\ (a \neq 0, b \neq 0)$ at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$.

Q.No : 17 Rate of Changes :

1. The distance – time formula for the motion of a particle along a straight line is $s = t^3 - 9t^2 + 24t - 18$. Find when and where the velocity is zero.
2. A point P is moving on the curve $y = 2x^2$. The x coordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y-coordinate is increasing when the point is at (2, 8).
3. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimeters ?
4. A container is in the shape of an inverted cone has height 8 cm and radius 6m at the top. If it is filled with water at the rate of $2m^3$/minute, how fast height of water changing when the level is 4m ?

5. A particle is moving in a straight line so that after t seconds its distance is s(in cms) from a fixed point on the line is given by $s = f(t) = 8t + t^3$.
Find i) the velocity at time t = 2 sec ii) the initial velocity iii) acceleration at t = 2 sec

6. The volume of a cube is increasing at the rate of 8 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm ?

7. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference.

8. A particle is moving along a line accor-ding to $s = f(t) = 4t^3 - 3t^2 + 5t - 1$ where s is measured in metres and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero.

9. The total cost c(x) in rupees associated with production of x units of an item is given by $c(x) = 0.005x^3 - 0.02x^2 + 30x + 500$. Find the marginal cost when 3 units are produced (marginal cost is the rate of change of total cost).

10. A stone is dropped into a quiet lake and ripples moves in circles at speed of 5 cm/sec. At the instant when the radius of circular ripple is 8 cm, how fast is the enclosed area increases ?

SECTION – C

III. Long Answer Type Questions : $5 \times 7 = 35$

i) Answer ANY FIVE questions.

ii) Each question carries SEVEN marks.

Q.No : 18 Straight Line :

1. Find the circumcentre of the triangle whose vertices are (–2, 3), (2, – 1) and (4, 0).
2. Find the circumcentre of the triangle whose vertices are (1, 3), (0, – 2) and (– 3, 1).
3. Find the orthocentre of the triangle with vertices (– 5, – 7), (13, 2) and (– 5, 6).
4. Find the orthocentre of the triangle with vertices (– 2, – 1), (6, – 1) and (2, 5).
5. Find the circumcentre of the triangle formed by the lines $x + y = 0$, $2x + y + 5 = 0$ and $x - y = 2$.
6. Find the circumcentre of the triangle formed by the lines $3x - y - 5 = 0$, $x + 2y - 4 = 0$ and $5x + 3y + 1 = 0$.
7. Find the orthocentre of the triangle formed by the lines $x + y + 10 = 0$, $x - y - 2 = 0$ and $2x + y - 7 = 0$.
8. Find the ortho centre of the triangle formed by the lines $7x + y - 10 = 0$, $x - 2y + 5 = 0$, $x + y + 2 = 0$.
9. Find the equations of the lines passing through (1, 2) and making an angle of 60° with the line $\sqrt{3}x + y + 2 = 0$.
10. Find the equation of the lines passing through (– 3, 2) and making an angle of 45° with the straight line $3x - y + 4 = 0$.
11. If Q(h, k) is the image of the point $p(x_1, y_1)$, with respect to the straight line $ax + by + c = 0$ then, $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$. Find the image of (1, – 2) with respect to the straight line.

Q.No : 19 Pair of Straight Lines :

1. Let the equation $ax^2 + 2hxy + by^2 = 0$ represent a pair of straight lines. Then the angle 'θ' between the lines is given by $\cos\theta = \frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$.
2. Prove that the product of the perpendicular from (α, β) to the pair of lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$.
3. Prove that the area of triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\frac{n^2\sqrt{h^2-ab}}{|am^2 - 2hlm + bl^2|}$ sq.units.
4. Show that the equation to the pair of bisectors of the angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$ is $h[x^2 - y^2] = (a - b)xy$.
5. If $s \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel lines, then $h^2 = ab$ and $bg^2 = af^2$. Also find the distance between the two parallel lines is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$ (or) $2\sqrt{\frac{f^2-bg}{b(a+b)}}$.
6. Prove that the equation $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ represents a pair of straight lines. Find the point of intersection. Also find the angle between them.
7. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines then prove that i) $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and ii) $h^2 \geq ab$, $g^2 \geq ac$ and $f^2 \geq bc$.
8. Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{|c|}{\sqrt{(a-b)^2+4h^2}}$.

Q.No : 20 Pair of Straight Lines :

1. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.
2. Let us find the lines joining the origin to the points of intersection of the curve $7x^2 - 4xy + 8y^2 + 2x - 4y - 8 = 0$ with the straight line $3x - y = 2$ and also the angle between them.
3. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2 + 2xy + y^2 + 2x + 2y - 5 = 0$ and the line $3x - y + 1 = 0$.
4. Find the value of k, if the lines joining the origin to the point of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.
5. Find the condition for the lines joining the origin to the points of intersection of $x^2 + y^2 = a^2$ and the line $lx + my = 1$ to coincide.
6. Find the condition for the chord $lx + my = 1$ of the circle $x^2 + y^2 = a^2$ (Whose Centre is the origin) to subtend a right angle at the origin.
7. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the line $6x - y + 8 = 0$ with the pair of straight lines. $3x^2 + 4xy - 4y^2 - 11x + 2y + 6 = 0$. Show that the lines so obtained make equal angles with the coordinate axes.

8. Show that two pairs of lines $6x^2 - 5xy - 6y^2 = 0$ and $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ from a square.

9. Show that two pairs of lines $3x^2 + 8xy - 3y^2 = 0$ and $3x^2 + 8xy - 3y^2 + 2x - 4y + 1 = 0$ from a square.

10. Find k, if the equation $2x^2 + kxy - 6y^2 + 3x + y + 1 = 0$ represents a pair of lines. Find the point of intersection of the lines and the angle between the lines for this value of k.

Q.No : 21 Direction Cosines And Direction Ratios :

1. If a ray makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, find $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$.
2. Find the angle between two diagonals of a cube.
3. The vertices of a triangle are A(1, 4, 2), B(– 2, 1, 2), C(2, 3, – 4). Find $\angle A, \angle B, \angle C$.
4. Find the direction cosines of two lines which are connected by the relations $l + m + n = 0$ and $mn - 2nl - 2lm = 0$.
5. Find the direction cosines of two lines which are connected by the relations $l - 5m + 3n = 0$ and $7l^2 + 5m^2 - 3n^2 = 0$.
6. Show that the lines whose direction cosines are given by $l + m + n = 0$, $2mn + 3nl - 5lm = 0$ are perpendicular to each other.
7. Find the angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$.
8. Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

Q.No : 22 Differentiation :

1. If $y = x\sqrt{a^2 + x^2} + a^2\log(x + \sqrt{a^2 + x^2})$ then show that $\frac{dy}{dx} = 2\sqrt{a^2 + x^2}$.
2. If $y = x^{\tan x} + (\sin x)^{\cos x}$, then find $\frac{dy}{dx}$.
3. If $y = (\sin x)^{\log x} + x^{\sin x}$ then find $\frac{dy}{dx}$.
4. If $x^y + y^x = a^b$ then show that $\frac{dy}{dx} = -\left[\frac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}\right]$.
5. If $f(x) = \sin^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}$ and $g(x) = \tan^{-1}\sqrt{\frac{x-\beta}{\alpha-x}}$ then show that $f'(x) = g'(x)x$, $(\beta < x < \alpha)$.
6. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.
7. If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right]$, then find $\frac{dy}{dx}$.
8. If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - \tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$ then show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.
9. Find the derivative of the function $x^x + (\cot x)^x$.
10. If $x^{\log y} = \log x$, then show that : $\frac{dy}{dx} = \frac{y}{x}\left(\frac{1 - \log x \log y}{(\log x)^2}\right)$

Q.No : 23 Tangent and Normal :

1. Show that the tangent at $P(x_1, y_1)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $y y_1^{-1/2} + x x_1^{-1/2} = a^{1/2}$.
2. If the tangent at any point P on the curve $x^m y^n = a^{m+n}$ $(mn \neq 0)$ meets the coordinate axes in A, B, then show that AP : BP is a constant.
3. At a point (x_1, y_1) on the curve $x^3 + y^3 = 3axy$, show that the tangent is $(x_1^2 - ay_1)x + (y_1^2 - ax_1)y = ax_1y_1$.
4. At any point 't' on the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$, find the lengths of tangent, normal, sub tangent and sub normal.
5. Find the lengths of sub tangent, sub normal at a point 't' on the curves $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$
6. Show that the curves $y^2 = 4(x + 1)$ and $y^2 = 36(9 - x)$ intersect orthogonally.
7. Show that the condition for the orthogonality of the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$.
8. Find the angle between the curves $y^2 = 4x$ and $x^2 + y^2 = 5$.
9. i) Find the angle between the curves $xy = 2$ and $x^2 + 4y = 0$. ii) Define angle between the curves.

Q.No : 24 Maxima And Minima :

1. If the curved surface of right circular cylinder inscribed in a sphere of radius 'r' is maximum. Show that the height of the cylinder is $\sqrt{2}r$.
2. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20 ft. Find the maximum area.
3. From a rectangular sheet of dimensions 30 cm × 80 cm. Four equal squares of side x cm are removed at the corners and the sides are then turned up so as to form an open rectangular box. Find the value of x so that the volume of the box is the greatest.
4. A wire of length l is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of pieces of the wire respectively so that the sum of the areas is the least ?
5. Find two positive integers x and y such that $x + y = 60$ and xy^3 is maximum.
6. Find two postive integers whose sum is 16 and the sum of whose square is minimum.
7. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.
8. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
9. The profit function P(x) of a company selling x items per day is given by $P(x) = (150 - x)x - 1000$. Find the no.of items that the company should manufacture to get maximum profit. Also find the maximum profit.

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MATHEMATICS MODEL PAPER

MATHEMATICS – II(A)

Time : 3 Hours] [Max. Marks : 75

Note : This question paper consists of **three** sections **A, B** and **C.**

SECTION – A

I. Very Short Answer Type Questions : **10 × 2 = 20**

i) Answer all the questions.

ii) Each question carries TWO marks.

Q.No. 1 : Complex Numbers

1. Write the complex number $(2 - 3i)(3 + 4i)$ in the form $A + iB$.
2. Write the complex number $(1 + 2i)^3$ in the form $A + iB$.
3. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$.
4. Write the conjugate of the complex number $(2 + 5i)(-4 + 6i)$.
5. Write the conjugate of the complex number $\frac{5i}{7+i}$.
6. Write the conjugate of the complex number $(3 + 4i)(2 - 3i)$.
7. Find the square root of $(-5 + 12i)$.
8. Find the square root of $7 + 24i$.
9. If $(a + ib)^2 = x + iy$, find $x^2 + y^2$.
10. Find the multiplicative inverse of $7 + 24i$.

Q.No. 2 : Complex Numbers

1. If $z = 2 - 3i$, then show that $z^2 - 4z + 13 = 0$.
2. Express the complex number in modulus - amplitude form $z = -\sqrt{7} + i\sqrt{21}$
3. Express the complex number in modulus - amplitude form $1 - i$.
4. Express the complex number in modulus - amplitude form $1 + i\sqrt{3}$.
5. If $z_1 = -1$ and $z_2 = -i$, then find Arg $(z_1 z_2)$.
6. If $z_1 = -1$, $z_2 = i$, then find Arg $\left(\frac{z_1}{z_2}\right)$.
7. If $z = x + iy$ and $|z| = 1$, find the locus of z.

8. If the amplitude of $(z-1)$ is $\frac{\pi}{2}$, find the locus of z.

9. If the Arg $\bar{z}_1$ and Arg $\bar{z}_2$ are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively, find (Arg z_1 + Arg z_2).

10. If $|z-3+i| = 4$, determine the locus of z.

Q.No. 3 : De Moivre's Theorem

1. Find all the values of $(1-i)^8$.

2. If $x = \text{cis}\,\theta$, then find the value of $\left(x^6 + \frac{1}{x^6}\right)$.

3. If A, B, C are the angles of a triangle such that $x = \text{cis}\,A$, $y = \text{cis}\,B$, $z = \text{cis}\,C$, then find xyz.

4. If α, β are the roots of the equation $x^2 + x + 1 = 0$, then prove that $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$.

5. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that $\frac{1}{2+\omega} + \frac{1}{1+2\omega} = \frac{1}{1+\omega}$.

6. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49$.

7. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that $(1-\omega+\omega^2)^6 + (1-\omega^2+\omega)^6 = 128$.

8. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that $(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega) = a^3+b^3$.

9. If $1, \omega, \omega^2$ are the cube roots of unity, then find the value of $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$.

10. Find the value of $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 - \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$.

Q.No. 4 : Quadratic Expressions

1. Find the quadratic equation whose roots are $\frac{p-q}{p+q}, -\frac{p+q}{p-q}$ $(p \neq \pm q)$.

2. Find the quadratic equation whose roots are $7 \pm 2\sqrt{5}$.

3. If α, β are roots of the equation $ax^2 + bx + c = 0$, find the value of $\alpha^2 + \beta^2$.

4. If α, β are roots of the equation $ax^2 + bx + c = 0$, find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

5. Find the nature of the roots of the equation without finding the roots $3x^2 + 7x + 2 = 0$.

6. If the equation $x^2 - 15 - m(2x-8) = 0$ has equal roots, find the value of m.

7. Prove that the roots of $(x-a)(x-b) = h^2$ are always real.

8. Find the maximum (or) minimum value of the quadratic expression $3x^2 + 2x + 11$.

9. For what values of x the expression $x^2 - 5x + 6$ is positive ?

10. For what values of x the expression $-6x^2 + 2x - 3$ is negative ?

11. If $x^2 - 6x + 5 = 0$ and $x^2 - 12x + p = 0$ have a common root, then find p.

Q.No. 5 : Theory of Equations

1. Form the monic polynomial equation of degree 3 whose roots are 2, 3 and 6.
2. If α, β, γ are the roots of $4x^3 - 6x^2 + 7x + 3 = 0$ then find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.
3. If 1, 1, α are the roots of $x^3 - 6x^2 + 9x - 4 = 0$, then find α.
4. If -1, 2 and α are the roots of $2x^3 + x^2 - 7x - 6 = 0$, then find α.
5. If 1, -2 and 3 are the roots of $x^3 - 2x^2 + ax + 6 = 0$, then find a.
6. If α, β and 1 are the roots of $x^3 - 2x^2 - 5x + 6 = 0$ then find α and β.
7. If 1, 2, 3 and 4 are the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ then find the values of a, b, c and d.
8. If the product of the roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9 then find a.
9. Find the algebraic equation whose roots 2 times the roots of $x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3 = 0$.
10. Find the polynomial equation whose roots are the reciprocals of the roots of the equation $x^4 + 3x^3 - 6x^2 + 2x - 4 = 0$.

Q.No. 6 : Permutations and Combinations

1. If ${}^nP_4 = 1680$, find n.
2. If ${}^nP_3 = 1320$, find n.
3. If ${}^nP_7 = 42 \cdot {}^nP_5$, find n.
4. If ${}^{12}P_5 + 5 \cdot {}^{12}P_4 = {}^{13}P_r$, find r.
5. Find the number of different chains that can be prepared using 7 different coloured beads.
6. Find the number of ways of preparing a chain with 6 different coloured beads.
7. Find the number of ways of arranging the letters of the word INDEPENDENCE.
8. Find the number of ways of arranging the letters of the word MATHEMATICS.
9. Find the number of ways of arranging the letters of the word INTERMEDIATE.
10. Find the number of ways of arranging the letters of the word SINGING.

Q.No. 7 : Permutations and Combinations

1. If ${}^nC_5 = {}^nC_6$ then find ${}^{13}C_n$.
2. Find the value of ${}^{10}C_5 + 2 \cdot {}^{10}C_4 + {}^{10}C_3$.
3. If ${}^{12}C_{s+1} = {}^{12}C_{2s-5}$, find s.
4. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r.
5. If ${}^{12}C_{r+1} = {}^{12}C_{3r-5}$, find r.
6. If ${}^9C_3 + {}^9C_5 = {}^{10}C_r$, then find r.
7. If ${}^nP_r = 5040$ and ${}^nC_r = 210$, find n and r.
8. If $10 \cdot {}^nC_2 = 3 \cdot {}^{n+1}C_3$, find n.
9. Find the number of ways of selecting 3 vowels and 2 consonants from the letters of the word EQUATION.
10. Find the number of ways of selecting 3 girls and 3 boys out of 7 girls and 6 boys.

Q.No. 8 : Binomial Theorem

1. Find the number of terms in the expansion of $(2x + 3y + z)^7$.
2. Write down and simplify 6^{th} term in $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$.
3. Write down and simplify 7^{th} term in $\left(\frac{4}{x^3}+\frac{x^2}{2}\right)^{14}$.
4. Find the middle terms in the expansion of $\left(\frac{3x}{7}-2y\right)^{10}$.
5. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1 + x)^{22}$, find the value of ${}^{13}C_r$.
6. Find the coefficient of x^{-7} in $\left(\frac{2x^2}{3}-\frac{5}{4x^5}\right)^7$.
7. Find the term independent of x in the expansion of $\left(\frac{3}{\sqrt[3]{x}}+5\sqrt{x}\right)^{25}$.
8. Find the numerically greatest term in the binomial expansion of $(1 - 5x)^{12}$, when $x = 2/3$.
9. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... a_{2n}x^{2n}$ then prove that $a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$.
10. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... a_{2n}x^{2n}$ then prove that $a_0 + a_3 + a_6 + a_9 + ... 3^{n-1}$.
11. Find the set of x for which the binomial expansion of $(3 - 4x)^{3/4}$ are valid.

Q.No. 9 : Measures of Dispersion

1. Find the mean deviation about the mean for the following data 3, 6, 10, 4, 9, 10.
2. Find the mean deviation about the median for the following data
   13, 17, 16, 11, 13, 16, 11, 18, 12, 17.
3. Find the mean deviation about the median for the following data 4, 6, 9, 3, 10, 13, 2.
4. Find the mean deviation from the mean of the following discrete data 6, 7, 10, 12, 13, 4, 12, 16.
5. Find the variance and standard deviation of the following data 5, 12, 3, 18, 6, 8, 2, 10.
6. Define the range for ungrouped data and also find the range of the given data
   38, 70, 48, 40, 42, 55, 63, 46, 54, 44.
7. The variance of 20 observations is 5. If each of the observations is multiplied by 2, find the variance of the resulting observations.
8. Find the mean deviation about the mean for the data : 38, 70, 48, 40, 42, 55, 63, 46, 54, 44
9. Find the mean deviation about the mean for the following distribution :

x_i	10	11	12	13
f_i	3	12	18	12

10. Find the mean deviation about the median for the following frequency distribution :

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Q.No. 10 : Random Variables and Probability Distributions

1. A poisson variable satisfies $P(X = 1) = P(X = 2)$, find $P(X = 5)$.
2. If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively.
3. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.
4. For a binomial distribution with mean 4 and variance 3, find n and p values also find $P(x \geq 1)$.
5. The probability that a person chosen at random is left handed (in hand writing) is 0.1 what is the probability that in a group of 10 people, there is one who is left handed.
6. If X is a poisson variate such that $P(X = 0) = P(X = 1) = k$ then show that $k = e^{-1}$.
7. If x is a poisson variate with $P(X = 2) = \frac{2}{3}P(X = 1)$, find $P(X = 0)$ and $P(X = 3)$.
8. If X is a poisson's variate such that $P(X = 1) = 3P(X = 2)$, then find the variance of X.
9. For a poisson variate X, $P(X = 2) = P(X = 3)$, find the variance of X.
10. Find the parameters of the binomial variate whose mean and variance are $\frac{15}{2}, \frac{15}{4}$ respectively.

SECTION - B

II. Short Answer Type Questions : **5 × 4 = 20**

i) Answer ANY FIVE the questions.

ii) Each question carries FOUR marks.

Q.No. 11 : Complex Numbers

1. If $z = 3 - 5i$ then show that $z^3 - 10z^2 + 58z - 136 = 0$.
2. If $z = x + iy$ and if the point P in the Argand plane represents z, then describe geometrically the locus of z satisfying the equation $|z - 2 - 3i| = 5$.
3. Determine the locus of z, $z \neq 2i$, such that $\text{Re}\left(\frac{z-4}{z-2i}\right) = 0$.
4. If $x + iy = \frac{1}{1 + \cos\theta + i\sin\theta}$ then, show that $4x^2 - 1 = 0$.
5. If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta}$ then, show that $x^2 + y^2 = 4x - 3$.
6. Show that the points in the Argand diagram represented by the complex numbers $2 + 2i$, $-2 - 2i$, $-2\sqrt{3} + 2\sqrt{3}i$ are the vertices of an equilateral triangle.
7. Show that the four points in the Argand plane represented by the complex numbers $2 + i$, $4 + 3i$, $2 + 5i$, $3i$ are vertices of a square.
8. Show that the points in the Argand plane represented by the complex numbers $-2 + 7i$, $\frac{-3}{2} + \frac{1}{2}i$, $4 - 3i$, $\frac{7}{2}(1 + i)$ are the vertices of a rhombus.

9. If the point P denotes the complex number $z = x + iy$ in the Argand plane and if $\frac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.

10. If $(x - iy)^{1/3} = a - ib$ then show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$.

11. If the real part of $\frac{z+1}{z+i}$ is 1, then find the locus of z.

Q.No. 12 : Quadratic Expressions

1. If x is real, prove that $\frac{x}{x^2-5x+9}$ lies between $\frac{-1}{11}$ and 1.

2. Prove that $\frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.

3. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x \in R$, then find the bounds for p.

4. If x is real, then find the range of $\frac{x^2+x+1}{x^2-x+1}$.

5. If x is real, then find the range of $\frac{x+2}{2x^2+3x+6}$.

6. Find the maximum value of the function $\frac{x^2+14x+9}{x^2+2x+3}$ over R.

7. If $c^2 \neq ab$ and the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal then show that $a^3 + b^3 + c^3 = 3abc$ (or) $a = 0$.

8. If x is real, then find the range of $\frac{2x^2-6x+5}{x^2-3x+2}$.

9. Show that none of the values of the function $\frac{x^2+34x-71}{x^2+2x-7}$ over R lies between 5 and 9.

10. Solve : $2x^4 + x^3 - 11x^2 + x + 2 = 0$

Q.No. 13 : Permutations and Combinations

1. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the words i) REMAST, ii) MASTER.
2. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order, find the rank of the word i) PRISON, ii) SIPRON.
3. Find the sum of all 4 digit numbers that can be formed using the digits 1, 3, 5 , 7 and 9 (without repetition.)
4. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order find the rank of the word EAMCET.

5. Find the number of numbers that are greater than 4000 which can be formed using the digits 0, 2, 4, 6, 8 without repeated.

6. Find the number of numbers less than 2000 that can be formed using the digits 1, 2, 3, 4 if repetition is allowed.

7. If the letters of the word RUBLE are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word 'LUBER'.

8. Find the number of ways of arranging the letters of the word ORGANIC so that

   i) all vowels come together, ii) no two vowels come together,

   iii) the relative positions of vowels and consonants are not disturbed.

9. Find the number of ways of arranging 6 boys and 6 girls in a row. In how many of these arrangements.

   i) All the girls are sit together, ii) no two girls are together, iii) boys and girls come alternative?

10. 9 different letters of an alphabet are given. Find the number of 4 letter words that can be formed using these 9 letters which have

    i) no letter is repeated, ii) atleast one letter is repeated.

Q.No. 14 : Permutations and Combinations

1. Simplify : $^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$

2. Simplify : $^{25}C_4 + \sum_{r=0}^{4} {}^{(29-r)}C_3$

3. Prove that $\frac{^{4n}C_{2n}}{^{2n}C_n} = \frac{1.3.5....(4n-1)}{\{1.3.5....(2n-1)\}^2}$

4. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

5. Find the number of ways of selecting 11 members cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the team contains atleast 4 bowlers and two wicket keepers.

6. Find the number of ways of forming a committee of 5 persons from a group of 5 Indians, and 4 Russians such that there are at least 3 Indians in the committee.

7. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans. So that always the Indians will be in majority in the committee.

8. A round table conference is attended by 3 Indians, 3 Chinese, 3 Canadians and 2 Americans. Find the number of ways of arranging them at the round table so that the delegates belonging to same country sit together.

9. Find the number of ways of seating 5 Indians, 4 Americans and 3 Russians at a round table so that i) all Indians sit together, ii) no two Russians sit together, iii) persons of same nationality sit together.

10. Prove that for $3 \le r \le n$, $^{n-3}C_r + 3.^{n-3}C_{r-1} + 3^{n-3}C_{r-2} + {}^{n-3}C_{r-3} = {}^{n}C_r$.

Q.No. 15 : Partial Fractions

1. Resolve the fraction into partial fraction $\frac{x^2-x+1}{(x+1)(x-1)^2}$.

2. Resolve the fraction into partial fraction $\frac{x^2+5x+7}{(x-3)^3}$.

3. Resolve the fraction into partial fraction $\frac{x^2-3}{(x+2)(x^2+1)}$.

4. Resolve the fraction into partial fraction $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$.

5. Resolve the fraction into partial fraction $\frac{2x^2+2x+1}{x^3+x^2}$.

6. Resolve the fraction into partial fraction $\frac{x^3}{(x-a)(x-b)(x-c)}$.

7. Resolve $\frac{x^4}{(x-1)(x-2)}$ into partial fractions.

8. Resolve the fraction into partial fraction $\frac{3x-18}{x^3(x+3)}$.

9. Resolve the fraction into partial fraction $\frac{2x^2+1}{x^3-1}$.

10. Find the coefficient of x^n in the power series expansion of $\frac{x-4}{x^2-5x+6}$ specifying the region in which the expansion is valid ?

Q.No. 16 : Probability

1. A number 'x' is drawn arbitrarily from the set {1, 2, ... 100}. Find the Probability that $x+\frac{100}{x}$ is greater than 29.
2. Find the probability of drawing an ace or a spade from a well - shuffled pack of 52 playing cards.
3. If A, B are two events with $P(A \cup B) = 0.65$, $P(AB) = 0.15$ then find the value of $P(A^C) + P(B^C)$.
4. A and B are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$. Find the probability that

   I) A does not occurs II) Neither A nor B occurs
5. A problem in Calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of them try independently.
6. i) Define conditional events conditional probability.

   ii) State and prove multiplication theorem on probability.

7. Suppose A and B are independent events with P(A) = 0.6, P(B) = 0.7 then compute

I) $P(A \cap B)$ II) $P(A \cup B)$ III) $P\left(\frac{B}{A}\right)$ IV) $P(A^C \cap B^C)$.

8. Let A and B be independent events with P(A) = 0.2, P(B) = 0.5.

Find I) $P\left(\frac{A}{B}\right)$ II) $P\left(\frac{B}{A}\right)$ III) $P(A \cap B)$ IV) $P(A \cup B)$.

9. If A, B, C are 3 independent events of an experiment such that $P(A \cap B^C \cap C^C) = \frac{1}{4}$, $P(A^C \cap B \cap C^C) = \frac{1}{8}$, $P(A^C \cap B \cap C^C) = \frac{1}{4}$ then P(A), P(B) and P(C).

10. A fair die is rolled. Consider the events A = {1, 3, 5}, B = {2, 3} and C = {2, 3, 4, 5}.

Find I) $P(A \cap B), P(A \cup B)$ II) $P\left(\frac{A}{B}\right), P\left(\frac{B}{A}\right)$ III) $P\left(\frac{A}{C}\right), P\left(\frac{C}{A}\right)$ IV) $P\left(\frac{B}{C}\right), P\left(\frac{C}{B}\right)$.

Q.No. 17 : Probability

1. If A and B are independent events of a random experiment then show that A^C and B^C are also independent.
2. In a committee of 25 members, each member is proficient either in mathematics or in statistics or in both. If 19 of these are proficient in mathematics, 16 in statistics, find the probability that a person selected from the committee is proficient in both.
3. The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get atleast one contract is $\frac{4}{5}$. Find the probability that he get both the contracts.
4. The probabilities of 3 mutually exclusive events are respectively given as $\frac{1+3P}{3}, \frac{1-P}{4}, \frac{1-2P}{2}$. Prove that $\frac{1}{3} \le p \le \frac{1}{2}$.
5. Two persons A and B are rolling a die on the condition that the person who gets 3 will win the game. If A starts the game, then find the probabilities of A and respectively to win the game.
6. If A, B, C are three events, show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.
7. A, B, C are 3 horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A, B and C to win the race ?
8. A, B, C are three newspapers from a city. 20% of the population read A, 16% read B, 14% read C, 8% both A and B, 5% both A and C, 4% both B and C and 2% all the three. Find the percentage of the population who read atleast one newspaper.
9. 'A' speaks truth in 75% of the cases and 'B' in 80% cases. What is the probability that their statements about an incident do not match ?
10. If one ticket is randomly selected from tickets numbered 1 to 30, then find the probability that the number on the ticket is i) a multiple of 5 or 7, ii) a multiple of 3 or 5.

SECTION – C

III. Long Answer Type Questions : **5 × 7 = 35**

i) Answer ANY FIVE questions.

ii) Each question carries SEVEN marks.

Q.No. 18 : De moivre's Theorem

1. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then show that

   i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$, ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

2. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, prove that

   $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

3. If n is an integer then show that

   $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

4. If n is an integer then show that $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1}\cos\left(\frac{n\pi}{3}\right)$.

5. If n is a positive integer, show that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}}\cos\left(\frac{n\pi}{4}\right)$.

6. Show that one value of $\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right) = -1$.

7. If n is a positive integer, show that $(P+iQ)^{1/n} + (P-iQ)^{1/n} = 2(P^2+Q^2)^{1/2n}\cos\left(\frac{1}{n}\tan^{-1}\frac{Q}{P}\right)$.

8. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in N$, show that

   $\alpha^n + \beta^n = 2^{n+1}\cos\left(\frac{n\pi}{3}\right)$.

9. Solve $(x-1)^n = x^n$, n is a positive integer.

10. Find all the roots of the equation $x^{11} - x^7 + x^4 - 1 = 0$.

Q.No. 19 : Theory of Equations

1. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots of this equation are in arithmetic progression.
2. Solve $8x^3 - 36x^2 - 18x + 81 = 0$, given that the roots are in A.P.
3. Solve $3x^3 - 26x^2 + 52x - 24 = 0$ the given, the roots are in Geometric Progression.
4. Solve $18x^3 + 81x^2 + 121x + 60 = 0$ given that one root is equal to half the sum of the remaining roots.
5. Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, given that the product of two of the roots is 6.
6. Find the polynomial equation whose roots are the translates of those of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ by -2.

7. Solve $x^4 - 4x^2 + 8x + 35 = 0$, given that $2 + i\sqrt{3}$ is a root.

8. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that $1 + i$ is one of its roots.

9. Solve $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$.

10. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

11. Solve $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$.

12. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

13. Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

14. Prove that the equation $x^5 - 5x^3 + 5x^2 - 1 = 0$ has three equal roots and find its root.

15. Find the polynomial equation whose roots are translates of the roots of the equation $x^5 - 4x^4 + 3x^2 - 4x + 6 = 0$ by -3.

Q.No. 20 : Binomial Theorem

1. If n is a positive integer and x is any non zero real number, then prove that

$$c_0 + c_1 \cdot \frac{x}{2} + c_2 \cdot \frac{x^2}{3} + c_3 \cdot \frac{x^3}{4} + \dots\dots\dots + c_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

2. For $r = 0, 1, 2, \dots\dots\dots n$, prove that $C_0.C_r + C_1.C_{r+1} + C_2.C_{r+2} + \dots\dots + C_{n-r}.C_n = {}^{2n}C_{n+r}$ and hence deduce that i) $C_0^2 + C_2^2 + \dots\dots + C_n^2 = {}^{2n}C_n$ ii) $C_0.C_1 + C_1.C_2 + \dots\dots + C_{n-1}C_n = {}^{2n}C_{n+1}$.

3. If 36, 84, 126 are three successive binomial coefficients in the expansion of $(1 + x)^n$, then find n.

4. If the 2nd, 3rd and 4th terms in the expansion of $(a + x)^n$ are respectively 240, 720, 1080, find a, x, n.

5. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the expansion of $(1 + x)^n$ are in A.P. then show that $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.

6. If the coefficients of x^9, x^{10}, x^{11} in the expansion of $(1 + x)^n$ are in A.P., then prove that $n^2 - 41n + 398 = 0$.

7. If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of $(x + a)^n$ then prove that i) $p^2 - Q^2 = (x^2 - a^2)^n$ ii) $4PQ = (x + a)^{2n} - (x - a)^{2n}$.

8. If the coefficients of 4 consecutive terms in the expansion of $(1 + x)^n$ are a_1, a_2, a_3, a_4 respectively then show that $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$.

9. Suppose that n is a natural number and I, F are respectively the integral part and fractional $= (7 + 4\sqrt{3})^n$ i) I is an odd integer ii) $(I + F)(I - F) = 1$.

10. If n is a positive integer, prove that $\sum_{r=1}^{n} r^3 \left(\frac{{}^nC_r}{{}^nC_{r-1}} \right)^2 = \frac{n(n+1)^2(n+2)}{12}$.

Q.No. 21 : Binomial Theorem

1. Find the sum of the infinite series $\frac{3.5}{5.10} + \frac{3.5.7}{5.10.15} + \frac{3.5.7.9}{5.10.15.20} + \dots\dots\dots + \infty$.

2. Find the sum of infinite terms of the series $\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1.3}{1.2}\cdot\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\cdot\frac{1}{10^6}+......\right)$.

3. If $x = \frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+........\infty$, then find $3x^2 + 6x$.

4. If $x = \frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+........$, then prove that $9x^2 + 24x = 11$.

5. Find the sum of the infinite series $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-........$

6. If $x = \frac{5}{(2!).3}+\frac{5.7}{(3!).3^2}+\frac{5.7.9}{(4!).3^3}+........$, then find the value of $x^2 + 4x$.

7. If $t = 1-\frac{1}{8}+\frac{1.3}{8.16}-\frac{1.3.5}{8.16.24}+.......\infty$, then prove that $5t^2 = 4$.

8. Find the sum of the infinite series $1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^2+........\infty$.

9. Find the sum of the infinite series $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+.........\infty$.

10. Find the sum of the infinite series $\frac{1}{4}+\frac{5}{4.8}+\frac{5.7}{4.8.12}+.......\infty$.

Q.No. 22 : Measures of Dispersion

1. Find the mean deviation about the mean for the following continuous distribution.

Sales (in Rs. thousand)	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Number of companies	5	15	25	30	20	5

2. Find the mean deviation about median for the following data.

x_i	6	9	3	12	15	13	21	22
f_i	4	5	3	2	5	4	4	3

3. Find the mean deviation about the mean for the given data using step deviation method.

Marks obtained	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	5	8	15	16	6

4. Calculate the variance and standard deviation of the following continuous frequency distribution.

Class interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency	3	7	12	15	8	3	2

5. Calculate the variance and standard deviation for a discrete frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

6. Find the mean deviation from the mean of the following data, using the step deviation method.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of students	6	5	8	15	7	6	3

7. Find the mean deviation about mean for the following data.

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

8. Find the mean deviation about the median for the following continuous distribution.

Marks obtained	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of boys	6	8	14	16	4	2

9. Find the mean deviation about the median for the following continuous distribution.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

10.

Age (years) (x_i)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of workers (f_i)	120	125	175	160	150	140	100	30

Q.No. 23 : Probability

1. State and prove addition theorem on probability.
2. State and prove Baye's theorem on probability,

   Statement : If $A_1, A_2, ..., A_n$ are mutually exclusive and exhaustive events in a sample space, S such that $P(A_i) > 0$ for $i = 1, 2, ..., n$ and E is any event with $P(E) > 0$ then $P\left(\frac{A_K}{E}\right) = \frac{P(A_K) . P\left(\frac{E}{A_K}\right)}{\sum_{i=1}^{n} P(A_i) P\left(\frac{E}{A_i}\right)}$

   for K = 1, 2, ..., n.
3. Suppose that an urn B_1 contains two white and 3 black balls and another urn B_2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, find the probability that the urn choosen was B_1.
4. Three boxes B_1, B_2 and B_3 contain balls with different colours as shown below.

	White	Black	Red
B_1	2	1	2
B_2	3	2	4
B_3	4	3	2

   A die is thrown. B_1 is chosen if either 1 or 2 turns up. B_2 is chosen if 3 or 4 turns up and B_3 is chosen if 5 or 6 turns up. Having chosen a box in this way, a ball is choosen at random from this box. If the ball drawn is found to be red, find the probability that it is drawn from box B_2.
5. Three urns have the following composition of balls :

   urn I : 1 white, 2 black

   urn II : 2 white, 1 black

   urn III : 2 white, 2 black

   One of the urns is selected at random and a ball is drawn. It turns out of two be white. Find the probability that it came from urn III.

6. Three boxes numbered I, II, III contain the balls as follows :

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from Box - II.

7. A, B, C are 3 newspapers from a city 20% of the population read A, 16% read B, 14% read C, 8% both A and B, 5% both A and C, 4% both B and C and 2% all the three. Find the percentage of the population who read atleast one newspaper.

8. A speaks truth in 75% of the cases and B in 80% cases. What is the probability that their statements about an incident do not match ?

9. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. If all of them fire at the same target, find the probability that i) only one of them hits the target, ii) atleast one of them hits the target.

10. A bag contains 12 two rupee coins, 7 one rupee coins and 4 half a rupee coins. If three coins are selected at random, then find the probability that,
    i) the sum of three coins is maximum
    ii) the sum of three coins is minimum
    iii) each coin is of different value.

Q.No. 24 : Random Variables and Probability Distributions

1. The range of a random variable X is {0, 1, 2}. Given that $P(X = 0) = 3c^3$, $P(X = 1) = 4c - 10c^2$, $P(X = 2) = 5c - 1$. Find (i) the value of c ii) $P(X < 1)$ iii) $P(1 < X \leq 2)$ iv) $P(0 < X \leq 3)$.

2.

X = x	−2	−1	0	1	2	3
P(X = x)	0.1	k	0.2	2k	0.3	k

is the probability distribution of a random variable X. Find the value of k and the variance of X.

3. A random variable X has the following probability distribution

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find i) the value of k ii) the mean and iii) $P(0 < X < 5)$.

4. The probability distribution of a random variable X is given below.

$X = x_i$	1	2	3	4	5
$P(X = x_i)$	k	2k	3k	4k	5k

Find the value of k and the mean and variance of X.

5. A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.

6. The range of a random variable X is (1, 2, 3, ...) and $P(X = k) = \frac{c^k}{k!}$ (k = 1, 2, 3, ...). Find the value of c and $P(0 < X < 3)$.

7. Find the constant c, so that $F(x) = c\left(\frac{2}{3}\right)^x$, x = 1, 2, 3, ... is the p.d.f. of a discrete random variable X.

8. Two dice are rolled at random. Find the probability distribution of the sum of the numbers on them. Find the mean of the random variable.

9. If X is a random variable with the probability distribution $P(X = k) = \frac{(k+1)c}{2^k}$, k = 0, 1, 2, 3, ... then find c.

10. In the experiment of tossing a coin n times, if the variable X denotes the number of heads and $P(X = 4)$, $P(X = 5)$, $P(X = 6)$ are in arithmetic progression then find n.

MATHEMATICS MODEL PAPER

MATHEMATICS – II B

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** sections **A, B** and **C.**

SECTION – A

I. Very Short Answer Type Questions : **10 × 2 = 20**

i) Answer ALL the questions.

ii) Each question carries TWO marks.

Q.No - 1 : Circle

1. Find the centre and radius of the circle $\sqrt{1+m^2}\,(x^2+y^2)-2cx-2mcy=0\ (c>0)$.
2. If $x^2+y^2+2gx+2fy-12=0$ represents a circle with centre (2, 3) find g, f and its radius.
3. Find the values of a, b if $ax^2+bxy+3y^2-5x+2y-3=0$ represents a circle. Also find the radius and centre of the circle.
4. Find the equation of the circle passing through the origin and having the centre at (– 4, – 3).
5. Find the equation of the circle passing through (2, 3) and concentric with the circle $x^2+y^2+8x+12y+15=0$
6. Find the other end of the diameter of the circle $x^2+y^2-8x-8y+27=0$ if one end of it is (2, 3)
7. If $x^2+y^2-4x+6y+c=0$ represents a circle with radius 6 then find the value of c.
8. Find the value of a if $2x^2+ay^2-3x+2y-1=0$ represents a circle and also find its radius.
9. Find the equation of the circle passing through (2, – 1) having the centre at (2, 3)
10. Find the equation of the circle which is concentric with $x^2+y^2-6x-4y-12=0$ and passing through (– 2, 14).

Q.No - 2 : Circle

1. If the length of the tangent from (2, 5) to the cirlce $x^2+y^2-5x+4y+k=0$ is $\sqrt{37}$ then find k.
2. If the length of the tangent from (5, 4) to the cirlce $x^2+y^2+2ky=0$ is 1 then find k.
3. Obtain the parametric equation of the circle $x^2+y^2-6x+4y-12=0$.
4. Obtain the parametric equation of the circle $(x-3)^2+(y-4)^2=8^2$.
5. State the necessary and sufficient condition for $lx+my+n=0$ to be a normal to the circle $x^2+y^2+2gx+2fy+c=0$.
6. Show that (4, – 2) and (3, – 6) are conjugate with respect to the circle $x^2+y^2-24=0$.
7. If (4, k) and (2, 3) are conjugate points with respect to the circle $x^2+y^2=17$ then find k.
8. Find the equation of the circle with centre (– 3, 4) and touching the Y - axis.
9. Find the equation of the circle with centre (2, 3) and touching the line $3x-4y+1=0$.
10. Find the length of the tangent from (12, 17) to the cirlce $x^2+y^2-6x-8y-25=0$.

Q.No - 3 : System of Circles

1. Find the angle between the circles $x^2+y^2-12x-6y+41=0$, $x^2+y^2+4x+6y-59=0$.
2. If the angle between the circles $x^2+y^2-12x-6y+41=0$ and $x^2+y^2+kx+6y-59=0$ is 45° find k.
3. Show that the circles $x^2+y^2-2x-2y=0$, $3x^2+3y^2-8x+29y=0$ intersect each other orthogonally.

4. Find k if the pair of circles $x^2 + y^2 - 6x - 8y + 12 = 0$, $x^2 + y^2 - 4x + 6y + k = 0$ are orthogonal.
5. Find the equation of the radical axis of the circles $x^2 + y^2 - 3x - 4y + 5 = 0$, $3(x^2 + y^2) - 7x + 8y - 11 = 0$.
6. Find the equation of the common tangent of the circles $x^2 + y^2 + 10x - 2y + 22 = 0$, $x^2 + y^2 + 2x - 8y + 8 = 0$ at their point of contact.
7. Find the equation of the common chord of the pair of circles $x^2 + y^2 - 4x - 4y + 3 = 0$, $x^2 + y^2 - 5x - 6y + 4 = 0$.
8. Find the equation of the radical axis of the circles $2x^2 + 2y^2 + 3x + 6y - 5 = 0$ and $3x^2 + 3y^2 - 7x + 8y - 11 = 0$.
9. Find the equation of the common chord of $(x-a)^2 + (y-b)^2 = c^2$, $(x-b)^2 + (y-a)^2 = c^2$.
10. Find k if the pair of circles $x^2 + y^2 - 5x - 14y - 34 = 0$, $x^2 + y^2 + 2x + 4y + k = 0$ are orthogonal.

Q.No - 4 : Parabola

1. Find the equation of the parabola whose vertex is $(3, -2)$ and focus is $(3, 1)$.
2. Find the equation of the parabola whose focus is $s(1, -7)$ and vertex is $A(1, -2)$.
3. Find the coordinates of the points on the parabola $y^2 = 8x$ whose focal distance is 10.
4. Find the coordinates of the points on the parabola $y^2 = 2x$. Whose focal distance is 5/2.
5. If $\left(\frac{1}{2}, 2\right)$ is one extremity of a focal chord of the parabola $y^2 = 8x$. Find the coordinates of the other extremity.
6. Find the equation of the tangent to the parabola $y^2 = 16x$ inclined at 60° to the X - axis.
7. Find the value of k in the line $2y = 5x + k$ is a tangent to the parabola $y^2 = 6x$.
8. Find the vertex and focus of the parabola $x^2 - 6x - 6y + 6 = 0$.
9. Find the equations of axis and directrix of the parabola $y^2 + 6y - 2x + 5 = 0$.
10. Find the coordinates of the vertex focus the equation of the directrix and axis of the parabola i) $y^2 = 16x$, ii) $x^2 = -4y$

Q.No - 5 : Hyperbola

1. If e, e_1 are the eccentricities of a hyperbola and its conjugate hyperbola prove that $\frac{1}{e_1} + \frac{1}{e_1^2} = 1$ to its asymptotes.
2. If the eccentricity of a hyperbola is 5/4, then find the eccentricity of its conjugate hyperbola.
3. Show that angle between the two asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\tan^{-1}\left(\frac{b}{a}\right)$ (or) $2\sec^{-1}(e)$.
4. If $3x - 4y + k = 0$ is a tangent to $x^2 - 4y^2 = 5$, find the value of k.
5. Find the eccentricity, length of the latusrectum of the hyperbola $4x^2 - 9y^2 = 27$.
6. Find the equations of the hyperbola whose foci are $(\pm 5, 0)$, the transverse axis is of length 8.
7. Find the product of lengths of the perpendiculars from any point on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ to its asymptotes.
8. If the angle between the asymptotes is 30° then find its eccentricity.
9. Find the equation to the hyperbola whose foci are $(4, 2)$ and $(8, 2)$ and eccentricity is 2.
10. Define rectangular hyperbola and find its eccentricity.

Q.No - 6 : Integration

1. Evaluate $\int \sec^2 x \operatorname{cosec}^2 x \, dx$.

2. Evaluate $\int \left(\frac{1}{\sqrt{1-x^2}} + \frac{2}{\sqrt{1+x^2}} \right) dx$

3. Evaluate $\int \frac{1}{x \log x [\log(\log x)]} dx$

4. Evaluate $\int \frac{\sin^4 x}{\cos^4 x} dx$

5. Evaluate $\int \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx$

6. Evaluate $\int \frac{e^x (1+x)}{\cos^2(xe^x)} dx$

7. Evaluate $\int \frac{x^8}{1+x^{18}} dx$

8. Evaluate $\int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx$

9. Evaluate $\int \sqrt{\sin x} \cos x \, dx$

10. Evaluate $\int \left(x + \frac{1}{x}\right)^3 dx \; x > 0$

Q.No - 7 : Integration

1. Evaluate $\int e^x (\tan x + \log \sec x) \, dx$

2. Evaluate $\int e^x \left(\frac{1 + x \log x}{x} \right) dx$

3. Evaluate $\int \frac{x e^x}{(x+1)^2} dx$

4. Evaluate $\int \frac{dx}{(x+5)\sqrt{x+4}}$

5. Evaluate $\int \frac{1}{\sqrt{4-9x^2}} dx$

6. Evaluate $\int x \log x \, dx$

7. Evaluate $\int \frac{dx}{(x+1)(x+2)}$

8. Evaluate $\int \sin^{-1} x \, dx$

9. Evaluate $\int \frac{1+\cos^2 x}{1-\cos^2 x} dx$

10. Evaluate $\int \frac{\sec^2 x}{(1+\tan x)^3} dx$

Q.No - 8 : Definite Integration

1. Evaluate $\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$

2. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$

3. Evaluate $\int_0^2 |1-x| \, dx$

4. Evaluate $\int_0^4 |2-x| \, dx$

5. Evaluate $\int_2^3 \frac{2x}{1+x^2} dx$

6. Evaluate $\int_0^{\pi} \sqrt{2+2\cos\theta} \, d\theta$

7. Evaluate $\int_0^1 \frac{x^2}{x^2+1} dx$

8. Evaluate $\int_0^4 \frac{x^2}{1+x} dx$

9. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x \, dx$

10. Evaluate $\int_0^{\pi/2} \cos^8 x \, dx$

Q.No - 9 : Definite Integration

1. Evaluate $\int_0^2 \sqrt{4-x^2} \, dx$

2. Evaluate $\int_0^{\pi/4} \sec^4 x \, dx$

3. Evaluate $\int_0^{\pi/2} \sin^2 x \, dx$

4. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$

5. Find the area bounded by the parabola $y = x^2$ the X - axis and the lines $x = -1, x = 2$.
6. Find the area cut off between the line $y = 0$ and the parabola $y = x^2 - 4x + 3$.
7. Find the area bounded between the curves $y^2 - 1 = 2x$ and $x = 0$.
8. Find the area of the region enclosed by the given curves $y = x^3 + 3, y = 0, x = -1, x = 2$.
9. Find the area of the region enclosed by the given curves $x = 4 - y^2, x = 0$.
10. Find the area of the region enclosed by the given curves $y = x^2, y = 3x$.

Q.No - 10 : Defferential Equations

1. Find the order and degree of the differential equation $\left[\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right]^{6/5} = 6y$
2. Find the order and degree of the differential equation $1+\left(\frac{d^2y}{dx^2}\right)^2 = \left[2+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$
3. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/3}$
4. Find the order and degree of the differential equation $\left[\left(\frac{dy}{dx}\right)^{1/2}+\left(\frac{d^2y}{dx^2}\right)^{1/3}\right]^{1/4} = 0$
5. Find the order and degree of the differential equation $x^{1/2}\left(\frac{d^2y}{dx^2}\right)^{1/3} + x\frac{dy}{dx} + y = 0$
6. Form the differential equation corresponding to $y = A \cos 3x + B \sin 3x$, where A and B are parameters.
7. From the differential equation of the family of all circles with their centres at the origin and also find its order.
8. Find the order of the differential equation corresponding to $y = c(x-c)^2$ where c is an arbitrary constant.
9. Find the general solution of $\frac{dy}{dx} = e^{x+y}$
10. Form the differential equation whose solution is $xy = ae^x + be^{-x}$, where a, b are arbitrary constants.

SECTION – B

II. Short Answer Type Questions : **5 × 4 = 20**

i) Answer ANY FIVE the questions.

ii) Each question carries FOUR marks.

Q.No - 11 : Circle

1. Find the equation of the circle whose centre lies on the X - axis and passing through (–2, 3) and (4, 5).
2. If a point P is moving such that the lengths of tangents drawn from P to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ are in the ratio 2 : 3 then find the equation of the locus of P.
3. Find the equation of the circles with centre (– 2, 3) and which cuts off a chord of length 2 units on line $3x + 4y + 4 = 0$.

4. Find the equations of the tangents to the circle $x^2 + y^2 + 2x - 2y - 3 = 0$ and perpendicular to $3x - y + 4 = 0$.
5. Show that $x + y + 1 = 0$ touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$ and find its point of contact.
6. Find the pole of the line $3x + 4y - 45 = 0$ with respect to the circle $x^2 + y^2 - 6x - 8y + 5 = 0$.
7. Find the value of K if the lines $x + y - 5 = 0$, $2x + ky - 8 = 0$ are conjugate w.r.t to the circle $x^2 + y^2 - 2x - 2y - 1 = 0$
8. Find the angle between the tangents drawn from (3, 2) to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$.
9. Find the pair of tangents drawn from (3, 2) to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$.
10. Find the inverse point of (– 2, 3) with respect of the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

Q.No - 12 : System of Circles

1. Find the equation of the circle which passes through the point (0, – 3) and intersects the circles given by the equations $x^2 + y^2 - 6x + 3y + 5 = 0$ and $x^2 + y^2 - x - 7y = 0$ orthogonally.
2. Find the equation of the circle which passes through the origin and intersects the circles $x^2 + y^2 - 4x + 6y + 10 = 0$, $x^2 + y^2 + 12y + 6 = 0$ orthogonally.
3. Find the equation of the circle which cuts the circles $x^2 + y^2 - 4x - 6y + 11 = 0$ and $x^2 + y^2 - 10x - 4y + 21 = 0$ orthogonally and has the diameter along the straight line $2x + 3y = 7$.
4. Find the equation of the circle which is orthogonal to each of the following three circles. $x^2 + y^2 + 2x + 17y + 4 = 0$, $x^2 + y^2 + 7x + 6y + 11 = 0$ and $x^2 + y^2 - x + 22y + 3 = 0$.
5. If $x + y = 3$ is the equation of the chord AB of the circle $x^2 + y^2 - 2x + 4y - 8 = 0$. Find the equation of the circle having AB as diameter.
6. Find the equation of the circle passing through the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$, $x^2 + y^2 - 2x - 15 = 0$ and (1, 2).
7. Find the radical centre of the circles $x^2 + y^2 + 4x - 7 = 0$, $2x^2 + 2y^2 + 3x + 5y - 9 = 0$, $x^2 + y^2 + y = 0$.
8. Find the radical centre of the three circles. $x^2 + y^2 - 4x - 6y + 5 = 0$, $x^2 + y^2 - 2x - 4y - 1 = 0$, $x^2 + y^2 - 6x - 2y = 0$
9. If the two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then show that $f'g = fg'$.
10. Find the equation and length of the common chord of the circles $x^2 + y^2 + 2x + 2y + 1 = 0$, $x^2 + y^2 + 4x + 3y + 2 = 0$

Q.No - 13 : Ellipse

1. Find the equation of the ellipse in the standard form whose distance between foci is '2' and the length of the latus rectum is 15/2.
2. Find the equation of the ellipse in the standard form such that distance between the foci is 8 and distance between the directrices is 32.
3. Find the value of 'k' if $4x + y + k = 0$ is tangent to the ellipse $x^2 + 3y^2 = 3$.
4. Find the eccentricity, coordinates of foci, length of latus rectum and equations of directrices of the ellipse $3x^2 + y^2 - 6x - 2y - 5 = 0$.
5. Find the eccentricity, co-ordinates of foci, length of latus rectum and equations of directrices of the ellipse $9x^2 + 16y^2 - 36x + 32y - 92 = 0$.
6. Find the length of major axis, minor axis, latus rectum, eccentricity, coordinates of centre, foci and the equations of directrices of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$.
7. Find the length of major axis, minor axis, latus rectum, eccentricity, coordinates of centre, foci and the equations of directrices of the ellipse $9x^2 + 16y^2 = 144$.
8. Find the equation of the ellipse with focus at (1, – 1), e = 2/3 and directrix as $x + y + 2 = 0$.
9. Find the equation of the ellipse in the standard form, if it passes through the points (– 2, 2) and (3, – 1).
10. Find the equation of tangent and normal to the ellipse $9x^2 + 16y^2 = 144$ at the end of the latus rectum in the 1st quadrant.

Q.No - 14 : Ellipse

1. Find the equations of tangents to the ellipse $2x^2 + 3y^2 = 11$ at the points whose ordinate is 1.
2. Find the equation of the tangents to the ellipse $2x^2 + y^2 = 8$ which are
   i) parallel to $x - 2y - 4 = 0$ ii) perpendicular to $x + y + 2 = 0$
   iii) which makes an angle $\frac{\pi}{4}$ with X - axis
3. If the normal at one end of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one end of the minor axis, then show that $e^4 + e^2 = 1$. (e is the eccentricity of the ellipse)
4. Show that the point of intersection of the ⊥r tangent to an ellipse lies on a circle.
5. The tangent and normal to the ellipse $x^2 + 4y^2 = 4$ at a point $P(\theta)$ on it meets the major axis in Q and R respectively. If $0 < \theta < \frac{\pi}{2}$ and QR = 2 then show that $\theta = \cos^{-1}(2/3)$.
6. Find the condition for the line $lx + my + n = 0$ to be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
7. Find the condition for the line $x \cos\alpha + y \sin\alpha = p$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
8. Show that the locus of the feet of the perpendiculars drawn from foci to any tangent of the ellipse is the auxiliary circle.
9. If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ meets its major axis and minor axis at M and N respectively, then prove that $\frac{a^2}{(CM)^2}+\frac{b^2}{(CN)^2}=1$ where c is the centre of the ellipse.
10. Find the equation of the ellipse referred to its major and minor axes as the coordinate axes X, Y respectively with latus rectum of length 4, and distance between foci $4\sqrt{2}$.

Q.No - 15 : Hyperbola

1. Find the centre, foci, eccentricity, equation of the directrices length of the latus rectum of the hyperbola $16y^2 - 9x^2 = 144$.
2. Find the centre, foci, eccentricity, equation of the directrices length of the latus rectum of the hyperbola $x^2 - 4y^2 = 4$.
3. Find the centre, eccentricity, foci, equation of directrices and the length of the latus rectum of the hyperbola $4(y + 3)^2 - 9(x - 2)^2 = 1$.
4. Find the equations of the tangents to the hyperbola $x^2 - 4y^2 = 4$ which are (i) parallel, (ii) perpendicular to the line $x + 2y = 0$.
5. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are (i) Parallel and (ii) Perpendicular to the line $y = x - 7$.
6. Prove that product of the perpendicular distance from any point on a hyperbola to its asymptotes is constant.
7. Prove that the point of intersection of two perpendicular tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ lies on the circle $x^2 + y^2 = a^2 - b^2$.
8. Find the equations of tangents drawn to the hyperbola $2x^2 - 3y^2 = 6$ through $(-2, 1)$.
9. Find the equations of normal to the hyperbola $x^2 - 3y^2 = 144$ at the end of the latus rectum (first quadrant).
10. If the line $lx + my + n = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then show that $a^2l^2 - b^2m^2 = n^2$.

Q.No - 16 : Definite Integration

1. Evaluate $\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$
2. Evaluate $\int_0^{\pi/2} \frac{\cos^{\frac{5}{2}} x}{\sin^{\frac{5}{2}} x + \cos^{\frac{5}{2}} x} dx$
3. Evaluate $\int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx$
4. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$
5. Evaluate : $\int_0^{\pi/4} \log(1 + \tan x)\, dx.$
6. Evaluate $\int_0^{\pi/2} \frac{dx}{4 + 5\cos x}$
7. Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$.
8. Find the area bounded between the curves $y = x^2 + 1$, $y = 2x - 2$, $x = -1$, $x = 2$.
9. Find the area bounded by the curves $y = \sin x$ and $y = \cos x$ between any two consecutive points of inter section.
10. Evaluate : $\int_0^4 (16 - x^2)^{\frac{5}{2}} dx$.

Q.No - 17 : Differential Equations

1. Solve $(xy^2 + x)\, dx + (yx^2 + y)dy = 0$
2. Solve $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$
3. Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$
4. Solve $\frac{dy}{dx} + y \sec x = \tan x$
5. Solve $\frac{dy}{dx} + y \tan x = \cos^3 x.$
6. Solve $\frac{dy}{dx} + y \tan x = \sin x.$
7. Solve $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$
8. Solve $(x + y + 1) \frac{dy}{dx} = 1$
9. Solve $\frac{dy}{dx} - y \tan x = e^x \sec x.$
10. Solve $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

SECTION – C

III. Long Answer Type Questions : **$5 \times 7 = 35$**

i) Answer ANY FIVE questions.

ii) Each question carries SEVEN marks.

Q.No - 18 : Circle

1. Find the equation of the circle passing through the points (1, 2), (3, – 4), (5, – 6).
2. Find the equation of the circle passing through the points (5, 7), (8, 1) and (1, 3).
3. Find the equation of the circle passing through the points (2, – 3), (– 4, 5) and having the centre on the line $4x + 3y + 1 = 0$.
4. Find the equation of the circle passing through the points (4, 1), (6, 5) and having the centre on the line $4x + y - 16 = 0$.
5. Find the equation of the circle passing through the points (4, 1), (6, 5) and having the centre on the line $4x + 3y - 24 = 0$.
6. Show that the points (1, 1), (–6, 0), (–2, 2), (–2, –8) are concyclic and find the equation of the circle on which they lie.

7. Find the value of c so that (2, 0), (0, 1), (4, 5) and (0, c) are concyclic.
8. Find the equation of the circle passing through (– 1, 0) and touching $x + y - 7 = 0$ at (3, 4).
9. Find the equations of circles passing through (1, – 1) touching the lines $4x + 3y + 5 = 0$ and $3x - 4y - 10 = 0$.
10. Find the equation of circles which touch $2x - 3y + 1 = 0$ at (1, 1) and having radius $\sqrt{13}$.

Q.No - 19 : Circle

1. Show that the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ touch each other. Find the point of contact and equation of the common tangent at the point of contact.
2. Show that the circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 8y + 13 = 0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.
3. Show that the circles $x^2 + y^2 - 6x - 9y + 13 = 0$ and $x^2 + y^2 - 2x - 16y = 0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.
4. Show that the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $5(x^2 + y^2) - 8x - 14y - 32 = 0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.
5. Find the equation to the direct common tangents to the circles $x^2 + y^2 + 22x - 4y - 100 = 0$, $x^2 + y^2 - 22x + 4y + 100 = 0$.
6. Find the transverse common tangents of the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$.
7. Find the equation of the circle which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at (5, 5) with radius 5.
8. Find the equation of the circle which touches $x^2 + y^2 - 4x + 6y - 12 = 0$ at (– 1, 1) internally with a radius of 2.
9. Prove that the equation to the pair of tangent to the circle s = 0 from $p(x_1, y_1)$ is $s_1^2 = s\, s_{11}$.
10. Find the locus of midpoints of the chords of contact of $x^2 + y^2 = a^2$ from the points on the line $lx + my + n = 0$

Q.No - 20 : Parabola

1. Prove that the equation of the parabola in standard form is $y^2 = 4ax$ (a > 0).
2. Find the equation of the parabola passing through the points (– 2, 1), (1, 2) and (– 1, 3) and having its axis parallel to X - axis.
3. Find the equation of the parabola passing through points (– 1, 2), (1, –1) and (2, 1) and having its axis parallel to the X - axis.
4. Find the equation of the parabola whose axis is parallel to Y - axis and which passes through the points (4, 5) (–2, 11) and (– 4, 21).
5. If y_1, y_2, y_3 are the y-coordinates of the vertices of the triangle inscribed in the parabola $y^2 = 4ax$ then show that the area of the triangle is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq. units.
6. Prove that the area of the triangle formed by the tangents at (x_1, y_1), (x_2, y_2) and (x_3, y_3) to the parabola $y^2 = 4ax$ (a > 0) is $\frac{1}{16a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq. units.
7. Show that the equation of common tangent to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$ are $y = \pm(x + 2a)$.
8. Show that the common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is $xa^{1/3} + yb^{1/3} + a^{2/3} \cdot b^{2/3} = 0$.
9. Find the equation of the parabola whose focus is (– 2, 3) and directrix is the line $2x + 3y - 4 = 0$. Also find the length of the latus - rectum and the equation of the axis of the parabola.
10. If $lx + my + n = 0$ is a normal to the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 + nm^2 = 0$.

Q.No - 21 : Integration

1. Evaluate $\int \frac{dx}{5+4\cos x}$

2. Evaluate $\int \frac{1}{4+5\sin x}dx$

3. Evaluate $\int \frac{1}{3\cos x+4\sin x+6}dx$

4. Evaluate $\int \frac{1}{4\cos x+3\sin x}dx$

5. Evaluate $\int \frac{1}{1+\cos x+\sin x}dx$

6. Evaluate $\int \frac{1}{\sin x+\sqrt{3}\cos x}dx$

7. Evaluate $\int \frac{2\cos x+3\sin x}{4\cos x+5\sin x}dx$

8. Evaluate $\int \frac{9\cos x-\sin x}{4\sin x+5\cos x}dx$

9. Evaluate $\int \frac{2\sin x+3\cos x+4}{3\sin x+4\cos x+5}dx$

10. Evaluate $\int \frac{\cos x+3\sin x+7}{\cos x+\sin x+1}dx$

Q.No - 22 : Integration

1. Evaluate $\int \frac{x+1}{x^2+3x+12}dx$

2. Evaluate $\int (6x+5)\sqrt{6-2x^2+x}\,dx$

3. Evaluate $\int (3x-2)\sqrt{2x^2-x+1}\,dx$

4. Evaluate $\int \frac{1}{(1+x)\sqrt{3+2x-x^2}}dx$

5. If $I_n = \int \sin^n x\,dx\ (n \geq 2)$ then prove that $I_n = \frac{-\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}I_{n-2}$. Hence find $\int \sin^4 x\,dx$.

6. Obtain reduction formula for $\int \cos^n x\,dx$ for $n \geq 2$ and deduce the value of $\int \cos^5 x\,dx$.

7. Obtain reduction formula for $I_n = \int \tan^n x\,dx$, n being a positive integer, $n \geq 2$ and deduce that the value of $\int \tan^6 x\,dx$.

8. Obtain reduction formula for $I_n = \int \cot^n x\,dx$, n being a positive integer, $n \geq 2$ and deduce that the value of $\int \cot^4 x\,dx$.

9. Obtain the reduction formula for $I_n = \int \sec^n x\,dx$, n being a positive integer, $n \geq 2$ and deduce that the value of $\int \sec^5 x\,dx$.

10. Obtain the reduction formula for $I_n = \int \operatorname{cosec}^n x\,dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \operatorname{cosec}^5 x\,dx$.

Q.No - 23 : Definite Integration

1. Evaluate $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

2. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

3. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

4. Evaluate $\int_0^{\pi} \frac{x\sin^3 x}{1+\cos^2 x}dx$

5. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x}dx$

6. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\sin x}dx$

7. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}dx$

8. Evaluate $\int_0^{\pi/2} \frac{x\,dx}{\sin x+\cos x}$

9. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Also deduce the area of circle $x^2+y^2=a^2$.

10. Find the area between the curves $y^2=4ax$, $x^2=4by$.

Q.No - 24 : Defferential Equations

1. Solve $(x^2+y^2)\,dx = 2xy\,dy$
2. Solve $(x^2-y^2)\,dx - xy\,dy = 0$
3. Solve $\frac{dy}{dx}=\frac{y^2-2xy}{x^2-xy}$
4. Solve $(x^2y-2xy^2)\,dx = (x^3-3x^2y)\,dy$
5. Solve $(x^3-3xy^2)\,dx + (3x^2y-y^3)\,dy = 0$
6. Solve $xy^2\,dy-(x^3+y^3)\,dx = 0$.
7. Find the solution of the equation $x(x-2)\frac{dy}{dx}-2(x-1)\,y = x^3(x-2)$, which satisfies the condition that $y = 9$ when $x = 3$.
8. Solve : $\frac{dy}{dx}=\frac{x-y+3}{2x-2y+5}$
9. Give the solution of $x\sin^2\frac{y}{x}dx = y\,dx - x\,dy$ which passes through the point $\left(1,\frac{\pi}{4}\right)$.
10. Find the equation of a curve whose gradient is $\frac{dy}{dx}=\frac{y}{x}-\cos^2\frac{y}{x}$, where $x>0$, $y>0$ and which passes through the point $\left(1,\frac{\pi}{4}\right)$.